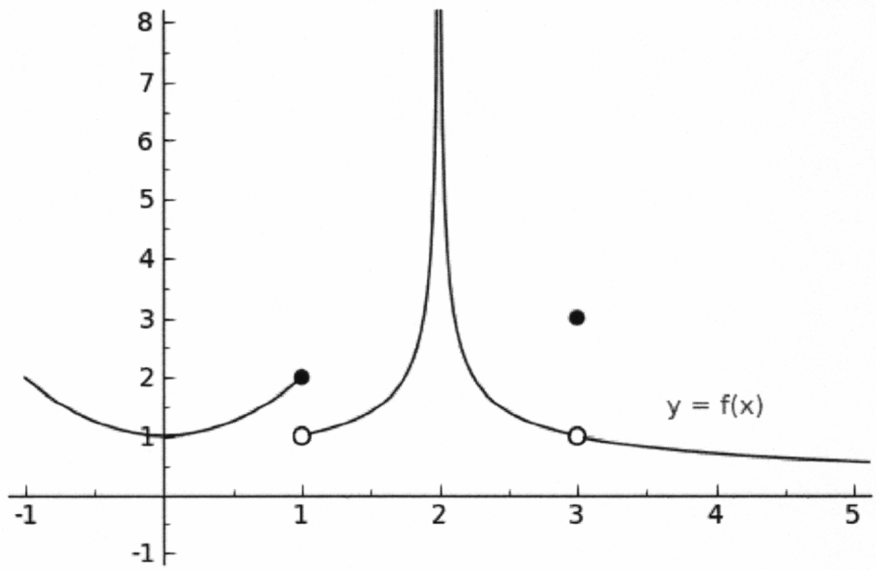
Name ______________________ Rec. Instr. ______________________
Signature ______________________ Rec. Time ______________________

Math 220
Exam 1
September 22, 2011

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		18	6		24
2		20	7		10
3		6	8		4
4		5	9		7
5		6	Total Score		100



1. (3 points each) Consider the graph of $y = f(x)$ above. State the value of each of the below quantities. If the limit does not exist, write "does not exist".

A. $\lim_{x \to 0} f(x)$ $= \boxed{1}$

B. $\lim_{x \to 1^-} f(x)$ $= \boxed{2}$

C. $\lim_{x \to 1^+} f(x)$ $= \boxed{1}$

D. $\lim_{x \to 1} f(x)$ does not exist

E. $\lim_{x \to 2} f(x)$ $=$ $+\infty$ (or does not exist)

F. $\lim_{x \to 3} f(x)$ $= \boxed{1}$

2. (5 points each) Evaluate the following limits. (Show your work.)

A. $\lim_{x\to 2} \frac{x-2}{x^2-4}$ $= \lim_{x\to 2} \frac{x-2}{(x-2)(x+2)} = \lim_{x\to 2} \frac{1}{x+2} = \boxed{\frac{1}{4}}$

B. $\lim_{t\to 2} \frac{\sqrt{2+t}-2}{t-2}$ $= \lim_{t\to 2} \frac{\sqrt{2+t}-2}{t-2} \cdot \frac{\sqrt{2+t}+2}{\sqrt{2+t}+2} = \lim_{t\to 2} \frac{(2+t)-4}{(t-2)(\sqrt{2+t}+2)}$

$= \lim_{t\to 2} \frac{t-2}{(t-2)(\sqrt{2+t}+2)} = \lim_{t\to 2} \frac{1}{\sqrt{2+t}+2} = \frac{1}{\sqrt{2+2}+2} = \boxed{\frac{1}{4}}$

C. $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$

For $x \neq 0$, $-x^2 \leq x^2 \sin(\frac{1}{x}) \leq x^2$.

$\lim_{x\to 0}(-x^2) = 0 = \lim_{x\to 0} x^2$

By the Squeeze Theorem, $\lim_{x\to 0} x^2 \sin(\frac{1}{x}) = \boxed{0}$

D. $\lim_{x\to -\infty} \frac{\sqrt{9x^2+4}}{2x+1}$ $= \lim_{x\to -\infty} \frac{\frac{\sqrt{9x^2+4}}{x}}{\frac{2x+1}{x}}$

For $x<0$, $x = -\sqrt{x^2}$

$= \lim_{x\to -\infty} \frac{\frac{\sqrt{9x^2+4}}{-\sqrt{x^2}}}{2+\frac{1}{x}}$

$= \lim_{x\to -\infty} \frac{-\sqrt{9+\frac{4}{x^2}}}{2+\frac{1}{x}} = \frac{-\sqrt{9+0}}{2+0} = \boxed{-\frac{3}{2}}$

3. (6 points) Is $f(x) = \begin{cases} 2 & \text{if } x = -1 \\ \frac{x^2-1}{x+1} & \text{if } x \neq -1 \end{cases}$ continuous at $x = -1$? (Explain your answer.)

$$\lim_{x \to -1} f(x) = \lim_{x \to -1} \frac{x^2-1}{x+1} = \lim_{x \to -1} \frac{(x-1)(x+1)}{x+1} = \lim_{x \to -1} (x-1) = -2$$

Since $\lim_{x \to -1} f(x) = -2 \neq 2 = f(-1)$, $f(x)$ is $\boxed{\text{discontinuous}}$ at $x = -1$

4. (5 points) A particle moves along a straight line with equation of motion $s(t) = 4\cos(t) + 2$, where $s(t)$ is measured in meters and t is measured in minutes. Using the limit definition of the derivative, set up (but do not evaluate) a formula for the velocity of the particle at time $t = 2$ minutes.

$$s'(2) = \lim_{h \to 0} \frac{s(2+h) - s(2)}{h} = \lim_{h \to 0} \frac{(4\cos(2+h)+2) - (4\cos(2)+2)}{h}$$

or

$$s'(2) = \lim_{y \to 2} \frac{s(y) - s(2)}{y-2} = \lim_{y \to 2} \frac{(4\cos(y)+2) - (4\cos(2)+2)}{y-2}$$

5. (6 points) Find the horizontal asymptote(s) for $y(x) = e^{-x} + 2$. (Show your work using limits.)

$$\lim_{x \to \infty} y(x) = \lim_{x \to \infty} (e^{-x}+2) = 2$$

$$\lim_{x \to -\infty} y(x) = \lim_{x \to -\infty} (e^{-x}+2) = +\infty$$

The only horizontal asymptote is $\boxed{y=2}$.

6. (6 points each) Calculate the derivative of the following functions. Use the rules for derivatives, not the limit definition of the derivative. You do not need to simplify your answer.

A. $w(x) = x^4 + \sqrt{x} + 1$

$$\boxed{w'(x) = 4x^3 + \frac{1}{2}x^{-1/2}}$$

B. $v(x) = e^x - \frac{1}{x}$

$$\boxed{v'(x) = e^x + x^{-2}}$$

C. $h(x) = x^3 e^x$

$$\boxed{h'(x) = 3x^2 e^x + x^3 e^x}$$

D. $g(x) = \frac{x^2-2}{x-1}$

$$\boxed{g'(x) = \frac{2x(x-1) - (x^2-2)\cdot 1}{(x-1)^2}}$$

7. (10 points) Find the tangent line to the curve $y = x^2$ at the point $(1,1)$. Use the limit definition of the derivative, and show your work.

$y = mx + b$

$$m = \lim_{h \to 0} \frac{(1+h)^2 - 1^2}{h} = \lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h} = \lim_{h \to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h \to 0} (2 + h) = 2$$

$y = 2x + b$

$1 = 2 \cdot 1 + b \Rightarrow b = 1 - 2 = -1$

$$\boxed{y = 2x - 1}$$

8. (4 points) Suppose that a waiter brings you a bowl of hot soup. Let $T(t)$ denote the temperature in degrees Fahrenheit of the soup after t minutes. Is $T'(3)$ positive or negative? (Explain your answer.)

$T'(3)$ is negative because the temperature of the soup is decreasing ~~3~~ 3 minutes after the waiter brings the hot soup.

9. (7 points) Sketch the graph of $y = w'(x)$ in the empty plot on the bottom of the page.