Name Solutions Rec. Instr.

Signature Rec. Time

Math 220

Exam 2

October 20, 2011

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work.**

Problem	Points	Points Possible	Problem	Points	Points Possible
1		30	6		10
2		5	7		10
3		10	8		9
4		5	9		5
5		8	10		8

Total Score:

1. (6 points each) Differentiate the following functions. You do not need to simplify your answers or show your work.

A. $w(x) = \ln(x) + 2^x$

$$w'(x) = \frac{1}{x} + 2^x \ln(2)$$

B. $v(x) = \dfrac{\cos(x)}{x^2 + 1}$

$$v'(x) = \frac{-\sin(x) \cdot (x^2+1) - \cos(x) \cdot (2x)}{(x^2+1)^2}$$

C. $h(x) = \left(\sin(\sqrt{x})\right)^5$

$$h'(x) = 5\left(\sin(\sqrt{x})\right)^4 \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

D. $p(x) = \tan^{-1}(x^2)$ (Recall that $\tan^{-1}(z) = \arctan(z)$.)

$$p'(x) = \frac{1}{1+(x^2)^2} \cdot 2x$$

E. $p(x) = (\tan(x) + x)^3$

$$p'(x) = 3(\tan(x)+x)^2 (\sec^2(x)+1)$$

2. (5 points) Evaluate $\lim_{x\to 1} \dfrac{\sin(x-1)}{x^2-1}$ $= \lim_{x\to 1} \dfrac{\sin(x-1)}{(x-1)(x+1)}$

$$= \left(\lim_{x\to 1} \frac{\sin(x-1)}{x-1}\right)\left(\lim_{x\to 1} \frac{1}{x+1}\right)$$

$$= \left(\lim_{y\to 0} \frac{\sin(y)}{y}\right) \cdot \frac{1}{2} = \frac{1}{2}$$

3. (5 points each)

A. Find the linearization (tangent line approximation) of $f(x) = \sqrt{x}$ at $x = 4$.

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$L(x) = f(4) + f'(4)(x-4) = 2 + \frac{1}{4}(x-4)$$

B. Use your answer from Part **A** to estimate $\sqrt{4.1}$.

4.1 is close to 4

$$\sqrt{4.1} = f(4.1) \approx L(4.1) = 2 + \frac{1}{4}(4.1-4) = 2.025$$

4. (5 points) The cost in dollars of producing x pounds of a chemical in a factory is given by $C(x) = 200 - x + 5x^2$. Find $C'(10)$, and interpret its meaning.

$C'(x) = -1 + 10x$ $\qquad$ $C'(10) = -1 + 10 \cdot 10 = 99$ \$/pound

This is the rate at which costs are increasing with respect to the number of pounds produced when $x = 10$ pounds.

(Alternatively, this is roughly the cost of producing the 11th pound of the chemical.)

5. (8 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = x^{\sin(x)}$.

$$\ln y = \ln\left(x^{\sin(x)}\right) = \sin(x)\ln(x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\left(\sin(x)\ln(x)\right) = \cos(x)\ln(x) + \sin(x)\frac{1}{x}$$

$$\frac{dy}{dx} = \left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right)y = \left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right)x^{\sin(x)}$$

6. (10 points) Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2 + xy + y^2 = 5$.

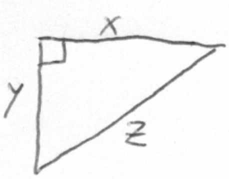
$$\frac{d}{dx}\left(x^2+xy+y^2\right) = \frac{d}{dx}5$$

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$(x+2y)\frac{dy}{dx} = -2x-y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

7. (10 points) Two people start walking from the same point. One person travels east at a rate of 4 miles per hour, and the other walks south at a rate of 3 miles per hour. At what rate is the distance between the two people changing 1 hour after they start walking? (Use related rates.)



x: dist traveled by person 1

y: dist traveled by person 2

z: dist between the people

Want: $\frac{dz}{dt}$ when $t = 1$ hr

Know: $\frac{dx}{dt} = 4$ mi/hr, $\frac{dy}{dt} = 3$ mi/hr

$$z^2 = x^2 + y^2$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

When $t = 1$ hr

$x = 4$ mi, $y = 3$ mi, $z = \sqrt{4^2+3^2} = 5$ mi

so $\frac{dz}{dt} = \frac{4\cdot 4 + 3\cdot 3}{5} = \frac{25}{5} = 5$ mi/hr

8. (3 points each) A stone is thrown vertically upward, and it's height in feet after t seconds is given by $s(t) = 64t - 16t^2$.

A. Find the velocity of the stone at time t seconds.

$$s'(t) = 64 - 32t$$

B. Over what time interval is the stone going upward?

$s'(t) > 0$ when $t < 2$ seconds

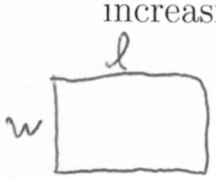
C. What is the maximum height the stone reaches?

$$s(2) = 64 \cdot 2 - 16 \cdot 2^2 = 64 \text{ ft}$$

9. (5 points) Find the differential dy if $y = (\ln(x))^2$.

$\frac{dy}{dx} = 2\ln(x) \cdot \frac{1}{x}$ so $dy = \frac{2\ln(x)}{x} \cdot dx$

10. (8 points) The length of a rectangle is increasing at a rate of 2 ft/s, and its width is increasing at a rate of 5 ft/s. At what rate is the area of the rectangle increasing when the length is 5 ft and the width is 10 ft.



Want $\frac{dA}{dt}$ when $l = 5\text{ft}$, $w = 10\text{ft}$

Know $\frac{dl}{dt} = 2\text{ ft/s}$, $\frac{dw}{dt} = 5\text{ ft/s}$

$A = l \cdot w$

$$\frac{dA}{dt} = \frac{dl}{dt} \cdot w + l \cdot \frac{dw}{dt}$$

when $l = 5\text{ft}$, $w = 10\text{ft}$, $\frac{dA}{dt} = 2 \cdot 10 + 5 \cdot 5 = 45 \text{ ft}^2/\text{s}$