Name Solutions Rec. Instr.

Signature Rec. Time

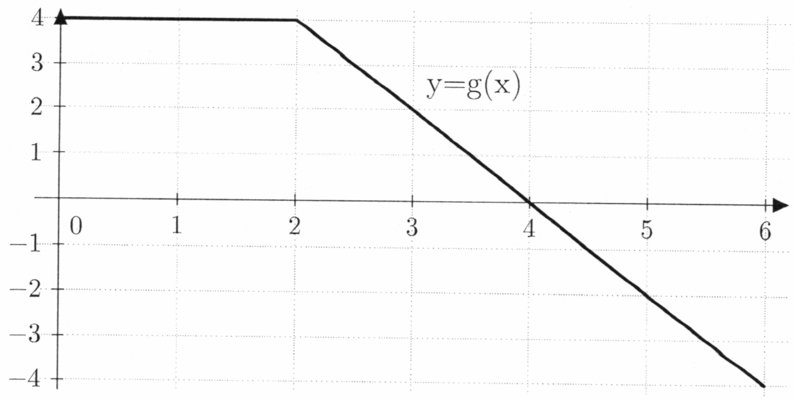
Math 220

Exam 3

November 17, 2011

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work.**

| Problem | Points | Points Possible | Problem | Points | Points Possible |
|---|---|---|---|---|---|
| 1 | | 12 | 6 | | 10 |
| 2 | | 10 | 7 | | 5 |
| 3 | | 10 | 8 | | 21 |
| 4 | | 12 | 9 | | 10 |
| 5 | | 10 | Total | | 100 |




**1.** (3 points each) The graph of $y = g(x)$ is shown above. Evaluate the following definite integrals. (You do not need to show your work.)

**A.** $\int_0^2 g(x)\,dx = 2 \cdot 4 = 8$

**B.** $\int_2^4 g(x)\,dx = \frac{1}{2} \cdot 2 \cdot 4 = 4$

**C.** $\int_4^6 g(x)\,dx = -\frac{1}{2} \cdot 2 \cdot 4 = -4$

**D.** $\int_0^6 g(x)\,dx = 8 + 4 - 4 = 8$

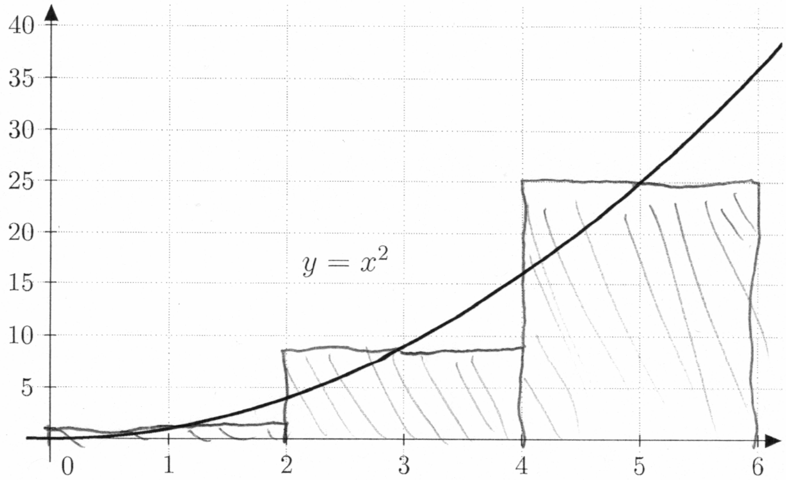
**2.** (10 points) Find $f(x)$ if $f''(x) = 6x + 4$, $f'(0) = 1$, and $f(0) = 2$.

$f'(x) = 3x^2 + 4x + C$

$1 = f'(0) = C$ so $f'(x) = 3x^2 + 4x + 1$

$f(x) = x^3 + 2x^2 + x + D$

$2 = f(0) = D$ so $f(x) = x^3 + 2x^2 + x + 2$




**3.** (10 points) Estimate $\int_0^6 x^2\,dx$ by using $n = 3$ subintervals, taking the sampling points to be midpoints. Also, illustrate the rectangles on the graph above.

$\Delta x = \frac{6-0}{3} = 2$

$\int_0^6 x^2\,dx \approx 2\cdot 1^2 + 2\cdot 3^2 + 2\cdot 5^2 = 2 + 18 + 50 = 70$

**4.** (4 points each) Find the most general antiderivative of the following functions. (I hope that you 'C' what I mean. You do not need to show your work.)

**A.** $e^x + \dfrac{1}{x^2}$

$e^x - \frac{1}{x} + C$

**B.** $\sec(x)\tan(x) + \sin(x)$

$\sec(x) - \cos(x) + C$

**C.** $4 + \dfrac{1}{1+x^2}$

$4x + \arctan(x) + C$

**5.** (10 points) Let $p(x) = 75 - x^2$ be the price in dollars per meal a chef can charge if he sells $x$ meals. ($p(x)$ is called the demand function.) What meal price will maximize revenue? (Recall, revenue is the total amount of money received from the sale of $x$ meals.)

Revenue: $R(x) = x \cdot p(x) = 75x - x^3$

We want to maximize $R(x)$ for $x > 0$

$R'(x) = 75 - 3x^2$

$R'(x) = 0 \Rightarrow 0 = 75 - 3x^2 \Rightarrow 3x^2 = 75 \Rightarrow x^2 = 25$

$\Rightarrow x = \pm 5 \Rightarrow x = 5$

$R''(x) = -6x < 0$ for $x > 0$

Hence, the absolute maximum for $R(x)$ occurs when $x = 5$ meals and $p(5) = 75 - 5^2 = 50$ \$/meal.

**6.** (10 points) Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 6x + 1$ on $[0, 3]$.

We can apply the closed interval method.

$f'(x) = 6x^2 - 6 = 6(x^2 - 1) = 6(x-1)(x+1)$.

Hence, our critical #'s are $x = \pm 1$, but $-1$ is not in $[0,3]$.

$f(0) = 1$

absolute min: $f(1) = 2 \cdot 1^3 - 6 \cdot 1 + 1 = -3$

absolute max: $f(3) = 2 \cdot 3^3 - 6 \cdot 3 + 1 = 54 - 18 + 1 = 37$

**7.** (5 points) Express the definite integral $\int_0^{\pi} \cos(x)\,dx$ as the limit as $n \to \infty$ of a Riemann sum. (Do not evaluate the limit.)

$\Delta x = \frac{\pi - 0}{n} = \frac{\pi}{n}$ $\qquad x_i = 0 + i\Delta x = \frac{\pi i}{n}$

$$\int_0^{\pi} \cos(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} \cos\left(\frac{\pi i}{n}\right) \cdot \frac{\pi}{n}$$

**8.** The function $f(x)$ and its first and second derivatives are:

$$f(x) = \frac{x^2}{x^2+3} \qquad f'(x) = \frac{6x}{(x^2+3)^2} \qquad f''(x) = \frac{-18(x^2-1)}{(x^2+3)^3}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE.

**A.** (1 point) Domain of $f(x)$: $(-\infty, \infty)$

**B.** (1 points) $y$-intercept: $(0,0)$ $x$-intercept(s): $(0,0)$

**C.** (1 points) Is $f(x)$ even or odd? even $f(-x) = \frac{(-x)^2}{(-x)^2+3} = \frac{x^2}{x^2+3} = f(x)$

**D.** (1 points) Vertical asymptote(s): None

**E.** (1 points) Horizontal asymptote(s): $y=1$

$\lim_{x\to\infty} \frac{x^2}{x^2+3} = \lim_{x\to\infty} \frac{1}{1+\frac{3}{x^2}} = 1$ $\lim_{x\to-\infty} \frac{x^2}{x^2+3} = \lim_{x\to-\infty} \frac{1}{1+\frac{3}{x^2}} = 1$

**F.** (2 points) Interval(s) $f(x)$ is increasing: $(0,\infty)$

(2 points) Interval(s) $f(x)$ is decreasing: $(-\infty, 0)$

$f'(x) > 0$ if $x > 0$

$f'(x) < 0$ if $x < 0$

**G.** (2 points) Local maximum(s)/minimum(s) $(x, y)$: local min at $(0,0)$ by 1st derivative test

**H.** (2 points) Interval(s) $f(x)$ is concave up: $(-1,1)$

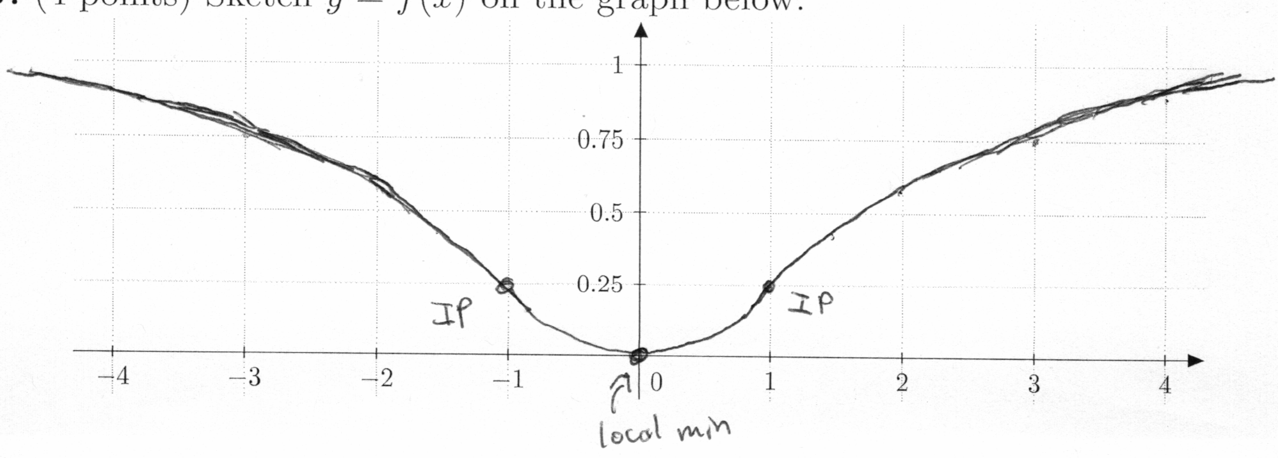
(2 points) Interval(s) $f(x)$ is concave down: $(-\infty,-1), (1,\infty)$

$f''(x) > 0 \iff x^2-1<0 \iff x^2<1 \iff -1<x<1$

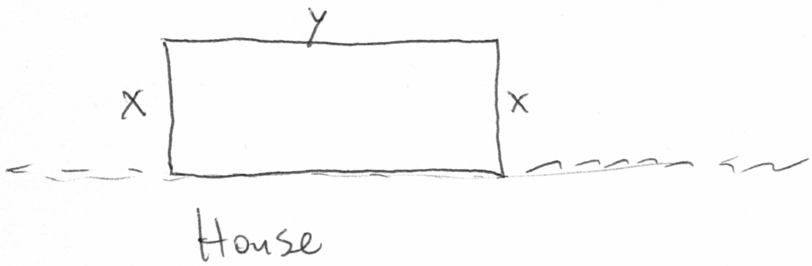
$f''(x) < 0 \iff x^2-1>0 \iff x^2>1 \iff x<-1$ or $x>1$

**I.** (2 points) Inflection point(s) $(x, y)$: $(-1,\frac{1}{4}), (1,\frac{1}{4})$

**J.** (4 points) Sketch $y = f(x)$ on the graph below.



**9.** (10 points) A homeowner with 16 feet of fencing wants to enclose a rectangular area against the side of her house. What dimensions will maximize the fenced-in area? (Note that three sides of the rectangle will be formed from fencing, and the house will serve as the fourth side of the rectangle. Make sure to justify why your answer corresponds to the absolute maximum.)




Maximize $A = xy$

$2x + y = 16 \Rightarrow y = 16 - 2x$

$A(x) = x(16-2x) = 16x - 2x^2$

$A'(x) = 16 - 4x$

$A'(x) = 0 \Rightarrow 0 = 16 - 4x \Rightarrow 4x = 16 \Rightarrow x = 4$

$A''(x) = -4$ so $A(x)$ is always concave down.

$A(x)$ is maximized when $x = 4$ feet and

$y = 16 - 2 \cdot 4 = 8$ feet.