Name Solutions Rec. Instr.

Signature Rec. Time

Math 220

Exam 1

February 2, 2012

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		7	5		8
2		10	6		8
3		24	7		8
4		35	Total Score		100

1. (7 points) Is $q(x) = \begin{cases} 4 & \text{if } x = 2 \\ \frac{x^2-4}{x-2} & \text{if } x \neq 2 \end{cases}$ continuous at $x = 2$? (Explain your answer.)

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} (x+2) = 4 = q(2)$$

Yes, $q(x)$ is continuous at $x=2$.

2. (10 points) Find the horizontal asymptote(s) for $y(x) = \dfrac{\sqrt{4x^2+1}}{3x+7}$. (Show your work using limits.)

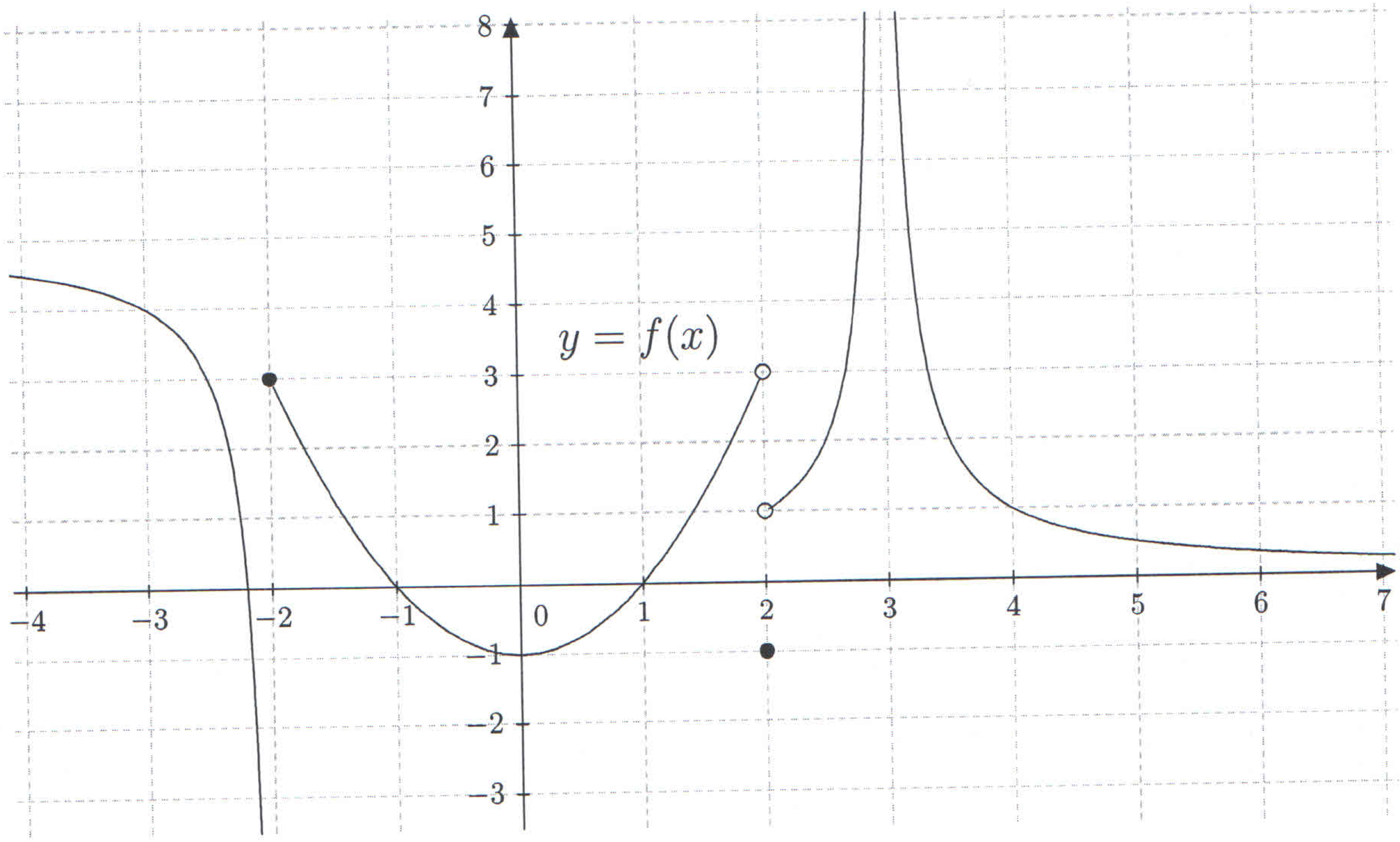
$$\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{3x+7} = \lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+1}}{x}}{\frac{3x+7}{x}} = \lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+1}}{\sqrt{x^2}}}{3+\frac{7}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{4+\frac{1}{x^2}}}{3+\frac{7}{x}} = \frac{\sqrt{4+0}}{3+0} = \frac{2}{3}$$

$$\lim_{x\to -\infty} \frac{\sqrt{4x^2+1}}{3x+7} = \lim_{x\to -\infty} \frac{\frac{\sqrt{4x^2+1}}{x}}{\frac{3x+7}{x}} = \lim_{x\to -\infty} \frac{\frac{\sqrt{4x^2+1}}{-\sqrt{x^2}}}{3+\frac{7}{x}}$$

$$= \lim_{x\to -\infty} \frac{-\sqrt{4+\frac{1}{x^2}}}{3+\frac{7}{x}} = -\frac{\sqrt{4+0}}{3+0} = -\frac{2}{3}$$

The horiz. asymptotes are $y=\frac{2}{3}$ and $y=-\frac{2}{3}$



3. (3 points each) Consider the graph of $y = f(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x\to 0} f(x) = -1$

E. $\lim_{x\to 2^-} f(x) = 3$

B. $\lim_{x\to -2^-} f(x) = -\infty$
(or does not exist)

F. $\lim_{x\to 2^+} f(x) = 1$

C. $\lim_{x\to -2^+} f(x) = 3$

G. $\lim_{x\to 2} f(x)$ does not exist

D. $\lim_{x\to 3} f(x) = +\infty$
(or does not exist)

H. $f(2) = -1$

4. (7 points each) Evaluate the following limits. (Show your work.)

A. $\lim_{x\to\pi} \sin(x) = \sin(\pi) = 0$ ($\sin(x)$ is continuous at $x=\pi$)

B. $\lim_{x\to 3} \frac{x-3}{x^2-5x+6} = \lim_{x\to 3} \frac{x-3}{(x-3)(x-2)} = \lim_{x\to 3} \frac{1}{x-2} = \frac{1}{3-2} = 1$

C. $\lim_{t\to 3} \frac{\sqrt{t+6}-3}{3-t} = \lim_{t\to 3} \frac{\sqrt{t+6}-3}{3-t} \cdot \frac{\sqrt{t+6}+3}{\sqrt{t+6}+3}$

$= \lim_{t\to 3} \frac{(t+6)-9}{(3-t)(\sqrt{t+6}+3)} = \lim_{t\to 3} \frac{t-3}{(3-t)(\sqrt{t+6}+3)}$

$= \lim_{t\to 3} \frac{-1}{\sqrt{t+6}+3} = \frac{-1}{\sqrt{3+6}+3} = -\frac{1}{6}$

D. $\lim_{x\to 2} (x-2)^2 \cos\left(\frac{\pi}{x-2}\right)$

For $x \neq 2$, $-(x-2)^2 \leq (x-2)^2 \cos\left(\frac{\pi}{x-2}\right) \leq (x-2)^2$

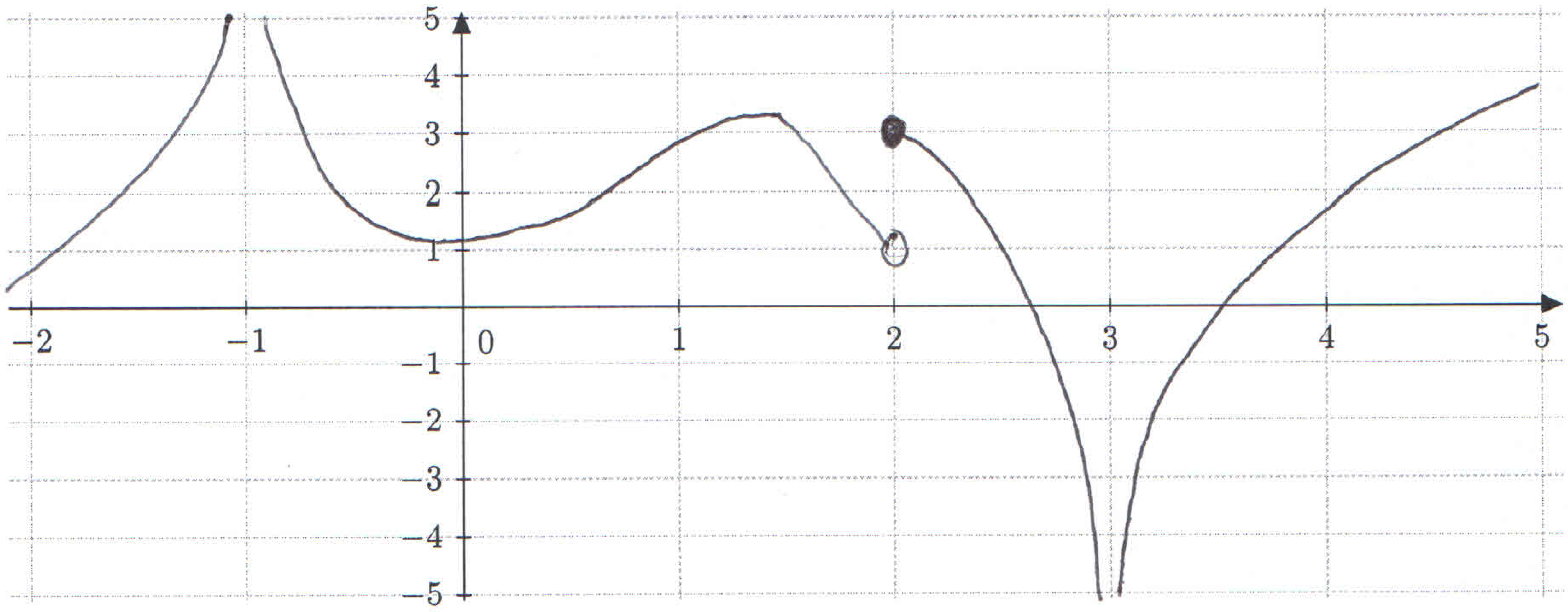
because $-1 \leq \cos\left(\frac{\pi}{x-2}\right) \leq 1$.

$\lim_{x\to 2} -(x-2)^2 = 0 = \lim_{x\to 2} (x-2)^2$.

By the Squeeze Theorem, $\lim_{x\to 2} (x-2)^2 \cos\left(\frac{\pi}{x-2}\right) = 0$

E. $\lim_{x\to\infty} \frac{2-e^x}{3e^x+5} = \lim_{x\to\infty} \frac{\frac{2-e^x}{e^x}}{\frac{3e^x+5}{e^x}} = \lim_{x\to\infty} \frac{2e^{-x}-1}{3+5e^{-x}} = \frac{0-1}{3+0} = -\frac{1}{3}$

5. (8 points) Sketch the graph of a function $v(x)$ that satisfies $\lim_{x\to -1} v(x) = \infty$, $\lim_{x\to 2^-} v(x) = 1$, $\lim_{x\to 2^+} v(x) = 3$, and $\lim_{x\to 3} v(x) = -\infty$.



6. (8 points) Use the Intermediate Value Theorem to show that there is a root of $x^5 + x - 1 = 0$ in the interval $(0, 1)$.

Let $f(x) = x^5 + x - 1$. $f(x)$ is continuous on $[0,1]$.

$f(0) = 0^5 + 0 - 1 = -1$. $f(1) = 1^5 + 1 - 1 = 1$. By the Intermediate Value Theorem, there exists a number c in $(0,1)$ with $f(c) = 0$, implying that $c^5 + c - 1 = 0$.

7. (4 points each) Given that $\lim_{x\to 1} h(x) = 2$ and $\lim_{x\to 1} w(x) = 3$, find the following limits.

A. $\lim_{x\to 1} (2h(x) + w(x)) = 2\left(\lim_{x\to 1} h(x)\right) + \left(\lim_{x\to 1} w(x)\right) = 2\cdot 2 + 3 = 7$

B. $\lim_{x\to 1} \dfrac{h(x)+2}{w(x)} = \dfrac{\left(\lim_{x\to 1} h(x)\right) + 2}{\left(\lim_{x\to 1} w(x)\right)} = \dfrac{2+2}{3} = \dfrac{4}{3}$

$\lim_{x\to 1} w(x) \neq 0$