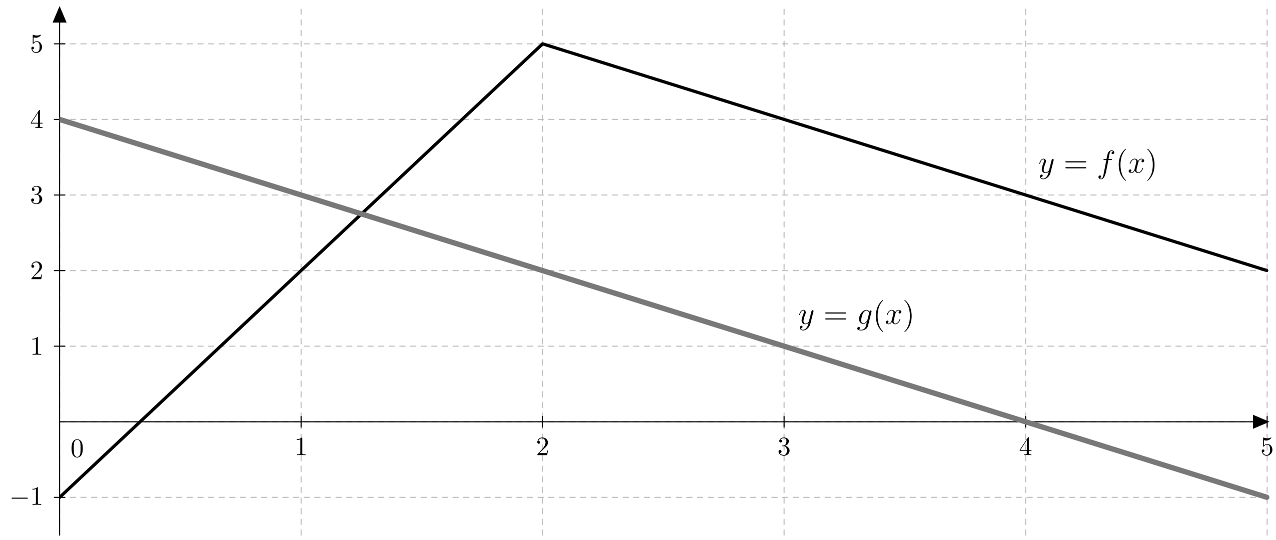
Name Solutions Rec. Instr.
Signature Rec. Time

Math 220
Exam 2
March 1, 2012

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		15	5		20
2		5	6		10
3		30	7		10
4		10	Total Score		100



1. (5 points each) For the graph above, calculate the following quantities.

A. $a'(1)$ if $a(x) = f(x)g(x)$

$$a'(x) = f'(x)g(x) + f(x)g'(x)$$

$$a'(1) = f'(1)g(1) + f(1)g'(1) = 3\cdot 3 + 2\cdot(-1) = 9-2 = \boxed{7}$$

B. $b'(1)$ if $b(x) = \dfrac{f(x)}{g(x)}$

$$b'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$b'(1) = \frac{f'(1)g(1) - f(1)g'(1)}{[g(1)]^2} = \frac{3\cdot 3 - 2(-1)}{3^2} = \frac{9+2}{9} = \boxed{\frac{11}{9}}$$

C. $c'(1)$ if $c(x) = f\,(g(x))$

$$c'(x) = f'(g(x))\cdot g'(x)$$

$$c'(1) = f'(g(1))\cdot g'(1) = f'(3)\cdot(-1) = (-1)\cdot(-1) = \boxed{1}$$

2. (5 points) Let $T(t)$ denote the temperature in degrees Fahrenheit of a cold can of soda t minutes after removing it from a refrigerator on a warm day. Is $T'(2)$ positive or negative? Explain your answer.

$T'(2)$ is positive because the can of soda's temperature is increasing 2 minutes after taking it out of the refrigerator.

3. (5 points each) Differentiate the following functions. You do not need to simplify your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $k(x) = 5x^3 + 3\sqrt{x} + \dfrac{2}{x}$

$$k'(x) = 5 \cdot 3x^2 + 3 \cdot \tfrac{1}{2} \cdot x^{-1/2} + 2 \cdot (-1) \cdot x^{-2}$$

$$= 15x^2 + \tfrac{3}{2}x^{-1/2} - 2x^{-2}$$

B. $w(x) = \ln(x) + 3^x$

$$w'(x) = \frac{1}{x} + 3^x \cdot \ln(3)$$

C. $v(x) = \cos(x)\sin(x)$

$$v'(x) = \left[\tfrac{d}{dx}\cos(x)\right]\sin(x) + \cos(x)\left[\tfrac{d}{dx}\sin(x)\right]$$

$$= -\sin(x) \cdot \sin(x) + \cos(x) \cdot \cos(x)$$

$$= -\sin^2(x) + \cos^2(x)$$

D. $p(x) = \dfrac{\tan(x)}{2e^x + 7}$

$$p'(x) = \frac{\left[\frac{d}{dx}\tan(x)\right](2e^x+7) - \tan(x)\left[\frac{d}{dx}(2e^x+7)\right]}{(2e^x+7)^2}$$

$$= \frac{\sec^2(x) \cdot (2e^x+7) - \tan(x) \cdot 2e^x}{(2e^x+7)^2}$$

E. $u(x) = \cos(\sec(x))$

$$u'(x) = -\sin(\sec(x)) \cdot \left[\tfrac{d}{dx}\sec(x)\right] = -\sin(\sec(x)) \cdot \sec(x) \cdot \tan(x)$$

F. $r(x) = x\tan^{-1}(x^3)$ (Recall that $\tan^{-1}(y) = \arctan(y)$.)

$$r'(x) = \left[\tfrac{d}{dx}x\right]\tan^{-1}(x^3) + x\left[\tfrac{d}{dx}\tan^{-1}(x^3)\right] = \tan^{-1}(x^3) + x \cdot \frac{1}{1+(x^3)^2}\left[\tfrac{d}{dx}x^3\right]$$

$$= \tan^{-1}(x^3) + \frac{x}{1+x^6} \cdot 3x^2 = \tan^{-1}(x^3) + \frac{3x^3}{1+x^6}$$

4. (10 points) Let $z(t) = 3t^2 + t$. Using the limit definition of the derivative, find $z'(1)$.

$$z'(1) = \lim_{h\to 0} \frac{z(1+h) - z(1)}{h} = \lim_{h\to 0} \frac{[3(1+h)^2 + (1+h)] - [3\cdot 1^2 + 1]}{h}$$

$$= \lim_{h\to 0} \frac{[3 + 6h + 3h^2 + 1 + h] - [3+1]}{h}$$

$$= \lim_{h\to 0} \frac{7h + 3h^2}{h} = \lim_{h\to 0} (7 + 3h) = 7 + 3\cdot 0 = \boxed{7}$$

5. (10 points each) Find $\frac{dy}{dx}$ if:

A. $x^3 + xy + y^4 = 5$

$$\frac{d}{dx}[x^3 + xy + y^4] = \frac{d}{dx} 5$$

$$\left[\frac{d}{dx} x^3\right] + \left[\frac{d}{dx} xy\right] + \left[\frac{d}{dx} y^4\right] = 0$$

$$3x^2 + \left[\frac{d}{dx} x\right] y + x\left[\frac{d}{dx} y\right] + 4y^3 \frac{dy}{dx} = 0$$

$$3x^2 + y + x\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 4y^3\frac{dy}{dx} = -3x^2 - y$$

$$\frac{dy}{dx}(x + 4y^3) = -3x^2 - y$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2 - y}{x + 4y^3}}$$

B. $y = (x^2+1)^5 x^{\sin(x)}$

$$\ln(y) = \ln[(x^2+1)^5 x^{\sin(x)}] = \ln[(x^2+1)^5] + \ln[x^{\sin(x)}] = 5\ln(x^2+1) + \sin(x)\ln(x)$$

$$\frac{d}{dx}\ln(y) = \frac{d}{dx}[5\ln(x^2+1) + \sin(x)\cdot\ln(x)] = 5\left[\frac{d}{dx}\ln(x^2+1)\right] + \left[\frac{d}{dx}\sin(x)\cdot\ln(x)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = 5\cdot\frac{1}{x^2+1}\left[\frac{d}{dx}(x^2+1)\right] + \left[\frac{d}{dx}\sin(x)\right]\ln(x) + \sin(x)\left[\frac{d}{dx}\ln(x)\right]$$

$$= \frac{5}{x^2+1}\cdot 2x + \cos(x)\cdot\ln(x) + \sin(x)\cdot\frac{1}{x} = \frac{10x}{x^2+1} + \cos(x)\cdot\ln(x) + \frac{\sin(x)}{x}$$

$$\frac{dy}{dx} = \left[\frac{10x}{x^2+1} + \cos(x)\cdot\ln(x) + \frac{\sin(x)}{x}\right]y = \boxed{\left[\frac{10x}{x^2+1} + \cos(x)\cdot\ln(x) + \frac{\sin(x)}{x}\right](x^2+1)^5 x^{\sin(x)}}$$

6. (10 points) Find the equation of the tangent line to the curve $y = e^{2x}$ at the point $(0, 1)$.

We want the tangent line $y = mx + b$.

Let $f(x) = e^{2x}$. By the Chain Rule, $f'(x) = 2e^{2x}$.

The slope is $m = f'(0) = 2e^{2\cdot 0} = 2e^0 = 2\cdot 1 = 2$.

Hence, our tangent line is of the form $y = 2x + b$.

Plugging in the point $(0,1)$, we get $1 = 2\cdot 0 + b = b$.

Our tangent line is $\boxed{y = 2x + 1}$.

7. (10 points) Sketch the graph of $y = q'(x)$ in the empty plot at the bottom of the page.