Name Solutions ______________ Rec. Instr. ______________
Signature ______________ Rec. Time ______________

Math 220
Exam 3
April 5, 2012

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		12	6		12
2		10	7		2
3		5	8		10
4		16	9		9
5		12	10		12

Total Score:

1. (12 points) Find the absolute maximum and absolute minimum of $h(x) = 2x^3 + 3x^2 - 12x + 1$ on $[0, 2]$.

$h'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1)$ is always defined so $h(x)$ is differentiable on $[0,2]$. $h'(x) = 0 \iff x = -2, 1$.
Hence, the only critical number for $h(x)$ on $[0,2]$ is at $x = 1$.

$h(0) = 2 \cdot 0^3 + 3 \cdot 0^2 - 12 \cdot 0 + 1 = 1$
$h(1) = 2 \cdot 1^3 + 3 \cdot 1^2 - 12 \cdot 1 + 1 = -6$
$h(2) = 2 \cdot 2^3 + 3 \cdot 2^2 - 12 \cdot 2 + 1 = 5$

For the interval $[0,2]$, $h(x)$ has an absolute maximum $(2,5)$ and an absolute minimum $(1,-6)$.

2. (5 points each)

A. Find the linearization (tangent line approximation) of $g(x) = e^x$ at $x = 0$.

$g'(x) = e^x$. The linearization of $g(x)$ at $x=0$ is
$L(x) = g(0) + g'(0)(x-0) = e^0 + e^0 \cdot x = 1 + x$.

B. Use your answer from Part **A** to estimate $e^{.01}$.

$e^{.01} = g(.01) \approx L(.01) = 1 + .01 = 1.01$

.01 is near 0

3. (5 points) Find dy if $y = \cos(x^2 + 1)$.

$\frac{dy}{dx} = -\sin(x^2+1) \cdot 2x = -2x\sin(x^2+1) \Rightarrow dy = -2x\sin(x^2+1) \cdot dx$

4. The function $f(x)$ and its first and second derivatives are:

$$f(x) = \frac{x}{\sqrt{x^2+1}} \qquad f'(x) = \frac{1}{(x^2+1)^{3/2}} \qquad f''(x) = \frac{-3x}{(x^2+1)^{5/2}}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: $(-\infty, \infty)$

B. (1 point) y-intercept: $f(0)=0$ so $(0,0)$ x-intercept(s): $f(x)=0 \Rightarrow x=0$ so $(0,0)$

C. (1 point) Is $f(x)$ even or odd? Odd $\quad f(-x) = \frac{-x}{\sqrt{(-x)^2+1}} = \frac{-x}{\sqrt{x^2+1}} = -f(x)$

D. (1 point) Vertical asymptote(s): None

E. (1 point) Horizontal asymptote(s): $y=1$ and $y=-1$

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{1}{\frac{\sqrt{x^2+1}}{x}} = \lim_{x\to\infty} \frac{1}{\frac{\sqrt{x^2+1}}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1+0}} = 1$$

$$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to-\infty} \frac{1}{\frac{\sqrt{x^2+1}}{x}} = \lim_{x\to-\infty} \frac{1}{\frac{\sqrt{x^2+1}}{-\sqrt{x^2}}} = \lim_{x\to-\infty} \frac{1}{-\sqrt{1+\frac{1}{x^2}}} = \frac{1}{-\sqrt{1+0}} = -1$$

F. (1 point) Interval(s) $f(x)$ is increasing: $(-\infty, \infty)$

(1 point) Interval(s) $f(x)$ is decreasing: None

For all x, $f'(x) = \frac{1}{(x^2+1)^{3/2}} > 0$

G. (1 point) Local maximum(s)/minimum(s) (x, y): None [There are no critical numbers]

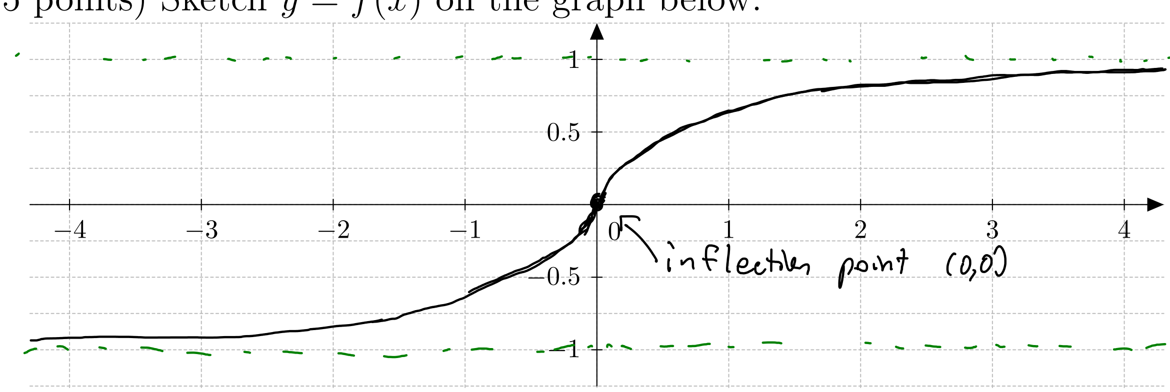
H. (1 point) Interval(s) $f(x)$ is concave up: $(-\infty, 0)$

(1 point) Interval(s) $f(x)$ is concave down: $(0, \infty)$

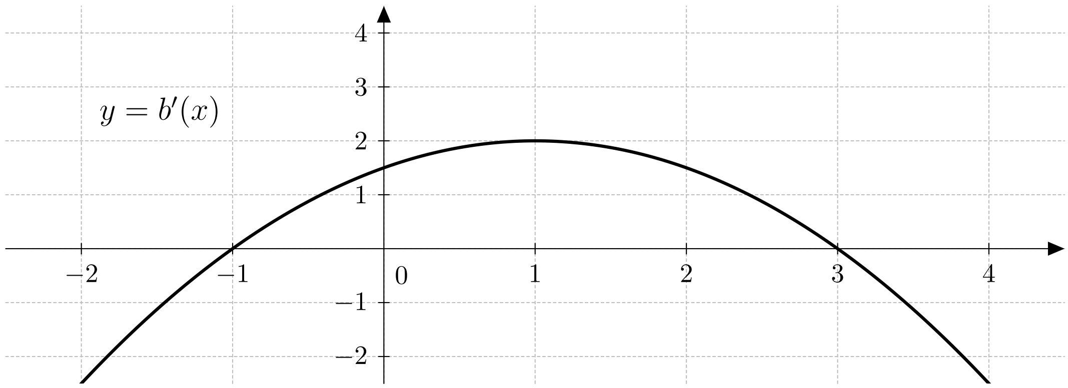
Since $(x^2+1)^{5/2}$ is always positive, $f''(x)$ has the same sign as $-3x$,
Hence, $f''(x) > 0$ on $(-\infty, 0)$, and $f''(x) < 0$ on $(0, \infty)$.

I. (1 point) Inflection point(s) (x, y): $(0,0)$

J. (5 points) Sketch $y = f(x)$ on the graph below.



inc / CU | inc / CD



5. (3 points each) $y = b'(x)$ is plotted above. Find:

A. Interval(s) where $b(x)$ is increasing: $(-1, 3)$ decreasing: $(-\infty, -1), (3, \infty)$

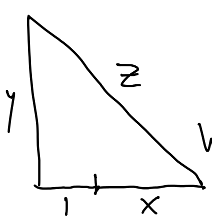
B. x-coordinate(s) where $b(x)$ has a local max: $x = 3$ local min: $x = -1$

C. Interval(s) where $b(x)$ is concave up: $(-\infty, 1)$ concave down: $(1, \infty)$

D. x-coordinate(s) where $b(x)$ has an inflection point: $x = 1$

6. (12 points) At noon, Ship A is 1 mile east of Ship B. Ship A is sailing east at 1 miles per hour, and Ship B is sailing north at 2 miles per hour. How fast is the distance between the ships changing at 2:00 PM.

x: dist Ship A has traveled t hours after noon
y: dist Ship B has traveled t hours after noon
z: dist between Ships A and B t hours after noon



We want $\frac{dz}{dt}$ when $t = 2$ hr. We know that $\frac{dx}{dt} = 1$ mi/hr and $\frac{dy}{dt} = 2$ mi/hr.

$$z^2 = (x+1)^2 + y^2$$

$$2z\frac{dz}{dt} = 2(x+1)\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{2(x+1)\frac{dx}{dt} + 2y\frac{dy}{dt}}{2z}$$

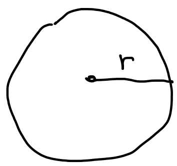
When $t = 2$ hr, $x = 2$ mi, and $y = 4$ mi. $z = \sqrt{(2+1)^2 + 4^2} = \sqrt{25} = 5$ mi.

Then, $\frac{dz}{dt} = \frac{2(2+1)\cdot 1 + 2\cdot 4\cdot 2}{2\cdot 5} = \frac{6 + 16}{10} = \frac{22}{10} = \frac{11}{5}$ mi/hr.

7. (2 points) Evaluate the expression 6+9/3. (No work needs to be shown.)

$$6 + 9/3 = 6 + \frac{9}{3} = 6 + 3 = 9$$

8. (10 points) The radius of a circle is increasing at a rate of 2 ft/s. At what rate is the area inside the circle increasing when the radius is 10 ft.



We want $\frac{dA}{dt}$ when $r = 10$ ft. We know that $\frac{dr}{dt} = 2$ ft/s.

$A = \pi r^2$ so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. When $r = 10$ ft,

$$\frac{dA}{dt} = 2\pi \cdot 10 \cdot 2 = 40\pi \text{ ft}^2/\text{s}.$$

9. (3 points each) A stone is thrown vertically upward, and its height in feet after t seconds is given by $s(t) = 32t - 16t^2$.

A. Find the velocity of the stone at time t seconds.

$$s'(t) = 32 - 32t = 32(1-t)$$

B. Over what time interval is the stone going upward?

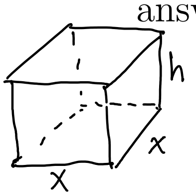
$$0 \text{ sec} \leq t < 1 \text{ sec}$$

C. What is the maximum height the stone reaches?

The stone reaches its maximum height at $t = 1$ sec.

$s(t) = 32 \cdot 1 - 16 \cdot 1^2 = 16$ ft.

10. (12 points) A rectangular open-topped box is to have a square base and volume 8 ft^3. If material for the base costs \$2 per ft^2 and material for the sides costs \$1 per ft^2, what dimensions minimize the cost of the box? (Justify why your answer is an absolute minimum.)



The cost is $C = \underbrace{1 \cdot 4xh}_{\text{Cost of 4 sides}} + \underbrace{2x^2}_{\text{cost of the base}}$

$8 = V = x^2 h \Rightarrow h = \frac{8}{x^2}$

$C(x) = 4x\left(\frac{8}{x^2}\right) + 2x^2 = \frac{32}{x} + 2x^2$. Minimize $C(x)$ on $(0, \infty)$.

$C'(x) = -\frac{32}{x^2} + 4x$ $\quad C'(x) = 0 \Rightarrow 4x = \frac{32}{x^2} \Rightarrow x^3 = \frac{32}{4} = 8 \Rightarrow x = 2$ ft.

$C''(x) = -32(-2)x^{-3} + 4 = \frac{64}{x^3} + 4$. $C''(x) > 0$ on $(0, \infty)$.

$C(x)$ is minimized when $x = 2$ ft and $h = \frac{8}{2^2} = 2$ ft.