Name Solutions Signature

Math 220 – Exam 1 – September 17, 2014

1. (4 points) Write an equation for the line with slope 5 that passes through the point $(3, 1)$.

$$y-1=5(x-3)$$

$$(\text{or } y=5x-14)$$

2. (5 points) Find $\displaystyle\lim_{x\to-\infty}\frac{6x^8+2x^3+1}{3x^8+4x^7+x^2}$.

$$= \lim_{x\to-\infty}\frac{\frac{6x^8+2x^3+1}{x^8}}{\frac{3x^8+4x^7+x^2}{x^8}}$$

$$= \lim_{x\to-\infty}\frac{6+\frac{2}{x^5}+\frac{1}{x^8}}{3+\frac{4}{x}+\frac{1}{x^6}}$$

$$= \frac{6+0+0}{3+0+0} = 2$$

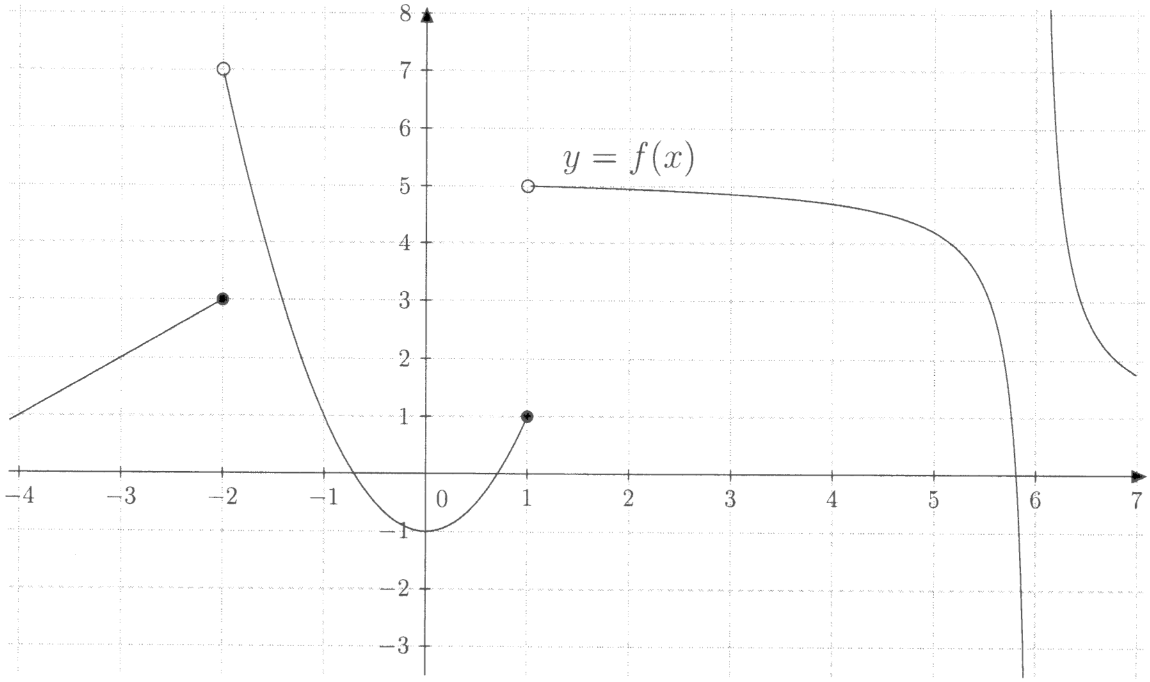
3. (7 points) Find the constant c that makes the following function continuous.

$$q(x) = \begin{cases} x^2+2 & \text{if } x > 2 \\ x+c & \text{if } x \le 2 \end{cases}$$

$q(x)$ is continuous on $(-\infty,2)$, $(2,\infty)$ no matter what c is.

$q(2)=2+c$. $\displaystyle\lim_{x\to2^-} q(x) = \lim_{x\to2^-}(x+c)=2+c$.

$\displaystyle\lim_{x\to2^+} q(x) = \lim_{x\to2^+}(x^2+2)=2^2+2=6$. Hence, for $q(x)$ to be continuous at $x=2$, we need $2+c=6$, making $c=4$.



4. (3 points each) Consider the graph of $y = f(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x \to -1} f(x) = 1$

E. $\lim_{x \to 1^-} f(x) = 1$

B. $\lim_{x \to -2^-} f(x) = 3$

F. $\lim_{x \to 1^+} f(x) = 5$

C. $\lim_{x \to -2^+} f(x) = 7$

G. $\lim_{x \to 1} f(x)$ does not exist

D. $\lim_{x \to 6^-} f(x) = -\infty$

H. $f(1) = 1$

5. (7 points each) Evaluate the following limits.

A. $\lim_{x\to 0} \frac{7\sin(x)}{x}$ $= 7 \lim_{x\to 0} \frac{\sin(x)}{x} = 7\cdot 1 = 7$

B. $\lim_{x\to 4} \frac{x-4}{x^2-5x+4}$ $= \lim_{x\to 4} \frac{x-4}{(x-4)(x-1)} = \lim_{x\to 4} \frac{1}{x-1} = \frac{1}{4-1} = \frac{1}{3}$

C. $\lim_{x\to 9} \frac{3-\sqrt{x}}{9-x}$ $= \lim_{x\to 9} \frac{3-\sqrt{x}}{9-x} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}} = \lim_{x\to 9} \frac{9-x}{(9-x)(3+\sqrt{x})}$

$= \lim_{x\to 9} \frac{1}{3+\sqrt{x}} = \frac{1}{3+\sqrt{9}} = \frac{1}{3+3} = \frac{1}{6}$

D. $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right)$

$-1 \le \cos\left(\frac{1}{x}\right) \le 1$ (for $x \neq 0$)

$-x^2 \le x^2 \cos\left(\frac{1}{x}\right) \le x^2$ (for $x \neq 0$)

$\lim_{x\to 0} (-x^2) = -0^2 = 0$. $\lim_{x\to 0} x^2 = 0^2 = 0$.

By the Squeeze Theorem, $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$.

x	1.9	1.99	1.999	1.9999	2.0001	2.001	2.01	2.1
$k(x)$	4.89	4.993	4.998	4.99992	5.00023	5.004	5.07	5.12

6. (4 points) Based on the table above, estimate $\lim_{x\to 2} k(x)$.

$$\lim_{x\to 2} k(x) = 5$$

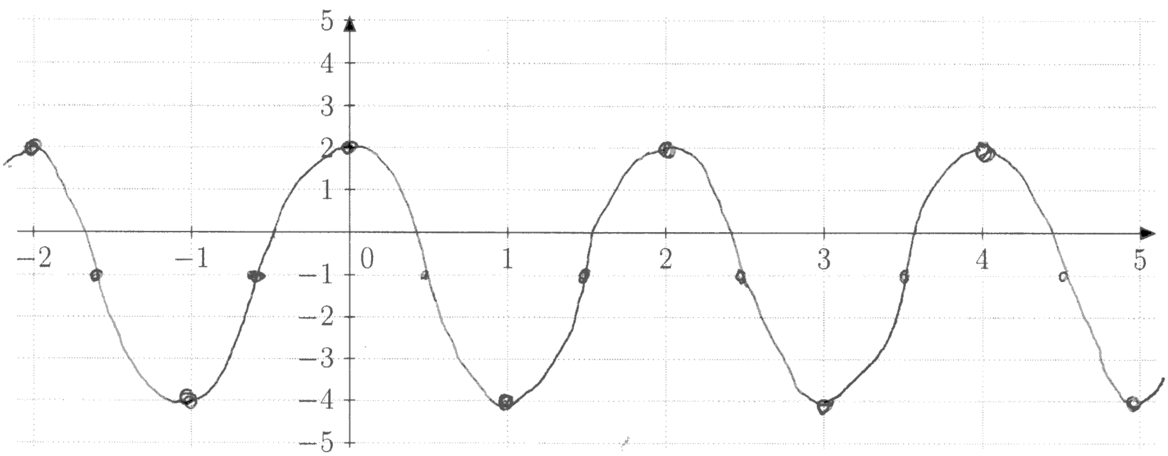
7. (7 points) Show that $x^7 + x^2 - 1 = 0$ has a solution in the interval $[0, 1]$.

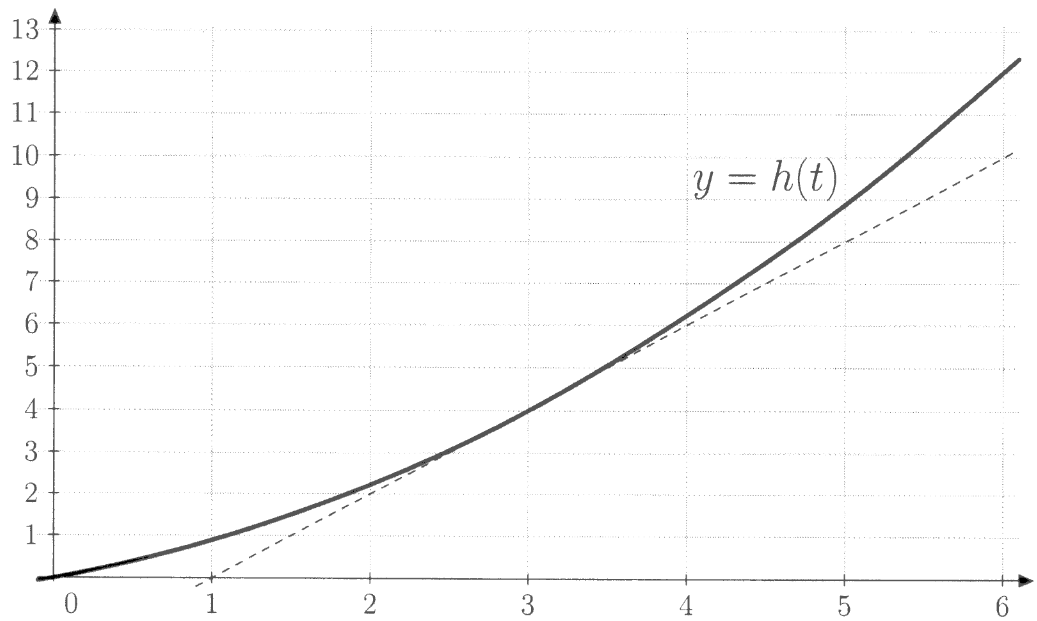
Let $f(x)=x^7+x^2-1$. $f(x)$ is continuous on $(-\infty,\infty)$.

$f(0)=0^7+0^2-1=-1$. $f(1)=1^7+1^2-1=1$.

By the Intermediate Value Theorem, there exists a c in $(0,1)$ with $0=f(c)=c^7+c^2-1$.

8. (7 points) Sketch the graph of $y = 3\cos(\pi x) - 1$.





9. (4 points each) A piece of paper is blowing in the wind. The function $h(t)$ graphed above denotes the height in feet of the paper after t seconds. The dotted line above is the tangent line to $y = h(t)$ at $t = 3$ seconds.

(a) Find the average velocity of the paper over the time interval $[3, 6]$ seconds.

$$\frac{h(6)-h(3)}{6-3} = \frac{12-4}{3} = \frac{8}{3} \frac{\text{ft}}{\text{sec}}$$

(b) Find the instantaneous velocity of the paper at time $t = 3$ seconds.

The slope of the tangent line to $y=h(t)$ at $t=3$ seconds is 2 ft/sec.

10. (6 points) Given that $\lim_{x\to 1} g(x) = 4$ and $\lim_{x\to 1} m(x) = 2$, find $\lim_{x\to 1} \dfrac{g(x)+x}{m(x)}$.

$$\lim_{x\to 1} \frac{g(x)+x}{m(x)} = \frac{\lim_{x\to 1} (g(x)+x)}{\lim_{x\to 1} m(x)} = \frac{\left[\lim_{x\to 1} g(x)\right] + \left[\lim_{x\to 1} x\right]}{2} = \frac{4+1}{2} = \frac{5}{2}$$