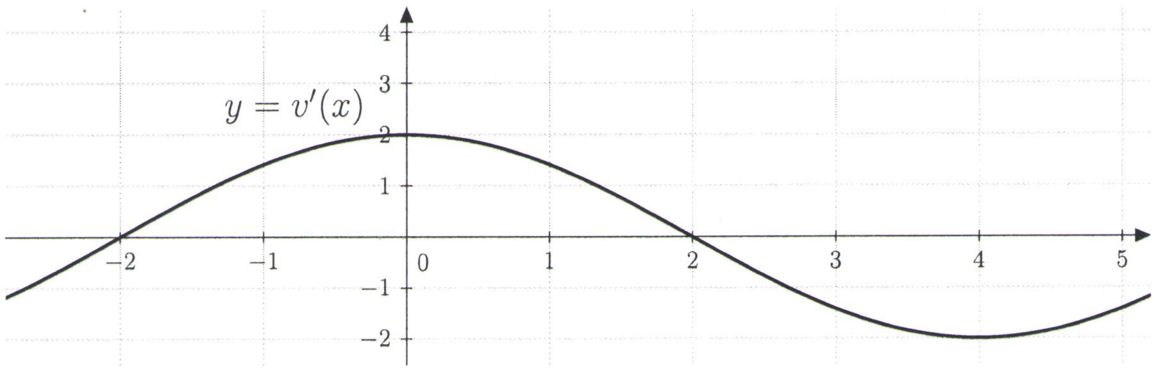
Name Solutions Signature

Math 220 – Exam 2 – October 15, 2014



1. (2 points each) $y = v'(x)$ is plotted above. Find the following:

A. Interval(s) where $v(x)$ is increasing: $(-2, 2)$

B. Interval(s) where $v(x)$ is decreasing: $(-\infty, -2), (2, \infty)$
(or $(-3,-2), (2,5)$)

2. (10 points) Let $q(t) = \frac{1}{t}$. Using the **limit definition of the derivative**, find $q'(3)$.

$$q'(3) = \lim_{h\to 0} \frac{q(3+h) - q(3)}{h} = \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3}{3(3+h)} - \frac{3+h}{3(3+h)}}{h} = \lim_{h\to 0} \frac{\frac{3-(3+h)}{3(3+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-h}{3(3+h)}}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{3(3+h)\cancel{h}} = \lim_{h\to 0} \frac{-1}{3(3+h)}$$

$$= \frac{-1}{3(3+0)} = -\frac{1}{9}$$

3. (4 points) Suppose that a waiter brings you a cold cup of iced tea. Let $F(t)$ denote the temperature in degrees Fahrenheit of the tea after t minutes. Is $F'(3)$ positive or negative? Explain your answer.

$F'(3)$ is positive because the temperature is increasing after 3 minutes. (The iced tea is heating up.)

4. (6 points) Find $\frac{d^2}{dx^2}5^x$.

$$\frac{d}{dx}5^x = 5^x \ln(5)$$

$$\frac{d^2}{dx^2}5^x = \frac{d}{dx}\left(5^x \ln(5)\right) = 5^x(\ln(5))^2$$

5. The height in feet of a ball t seconds after being thrown is given by $h(t) = -16t^2 + 10t + 6$.

A. (4 points) Find the velocity 1 second after the ball is thrown.

$$v(t) = h'(t) = -32t + 10$$

$$v(1) = -32(1) + 10 = -22 \text{ ft/sec}$$

B. (2 points) Is the ball going upward or downward 1 second after being thrown?

downward

6. (9 points) Let $w(x) = x^{\cos(x)}(x^3+x)^5$. Find $w'(x)$.

$$\ln(w(x)) = \ln(x^{\cos(x)}(x^3+x)^5) = \ln(x^{\cos(x)}) + \ln((x^3+x)^5)$$

$$\ln(w(x)) = \cos(x)\cdot\ln(x) + 5\ln(x^3+x)$$

$$\frac{w'(x)}{w(x)} = \frac{d}{dx}\ln(w(x)) = -\sin(x)\ln(x) + \cos(x)\cdot\frac{1}{x} + 5\cdot\frac{3x^2+1}{x^3+x}$$

$$w'(x) = \left(-\sin(x)\ln(x) + \frac{\cos(x)}{x} + \frac{15x^2+5}{x^3+x}\right) w(x)$$

$$w'(x) = \left(-\sin(x)\ln(x) + \frac{\cos(x)}{x} + \frac{15x^2+5}{x^3+x}\right) x^{\cos(x)}(x^3+x)^5$$

7. (9 points) Find $\frac{dy}{dx}$ for $x^4 + xy + 2y^3 = 5$.

$$\frac{d}{dx}[x^4 + xy + 2y^3] = \frac{d}{dx}5$$

$$4x^3 + 1\cdot y + x\cdot\frac{dy}{dx} + 6y^2\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = -4x^3 - y$$

$$(x+6y^2)\frac{dy}{dx} = -4x^3 - y$$

$$\frac{dy}{dx} = \frac{-4x^3-y}{x+6y^2}$$

8. (7 points each) Find the following derivatives. You do not need to simplify.

A. $\frac{d}{dx}(\cos(e^x)) = -\sin(e^x)\cdot\left[\frac{d}{dx}e^x\right] = -\sin(e^x)\cdot e^x$

B. $\frac{d}{dx}(\ln(x)\cdot\arctan(x)) = \frac{1}{x}\cdot\arctan(x) + \ln(x)\cdot\frac{1}{1+x^2}$

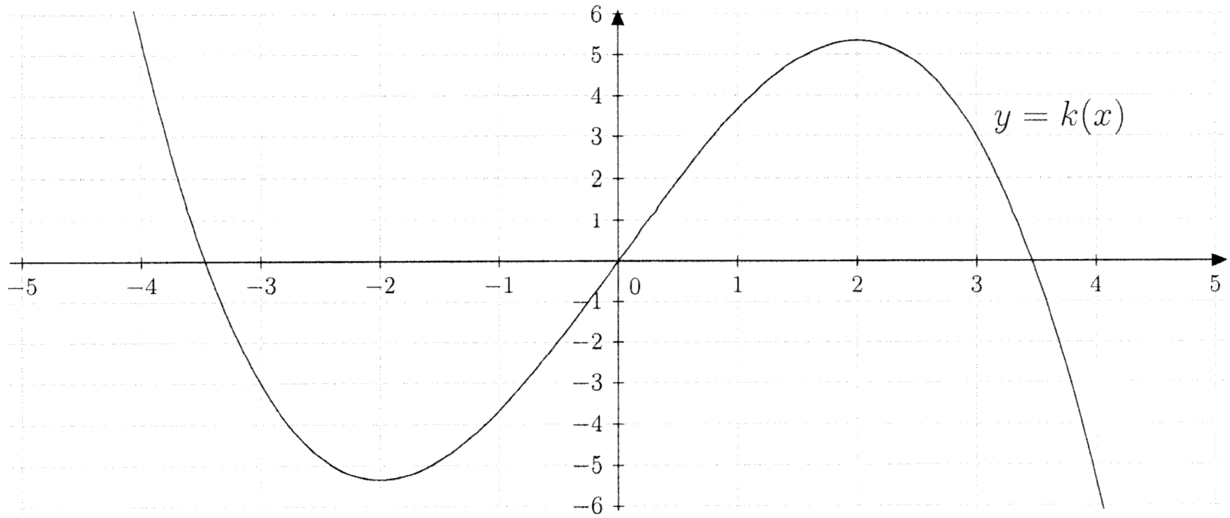
C. $\frac{d}{dx}\left(\frac{\tan(x)}{x^8+x^4}\right) = \frac{\sec^2(x)(x^8+x^4) - \tan(x)(8x^7+4x^3)}{(x^8+x^4)^2}$

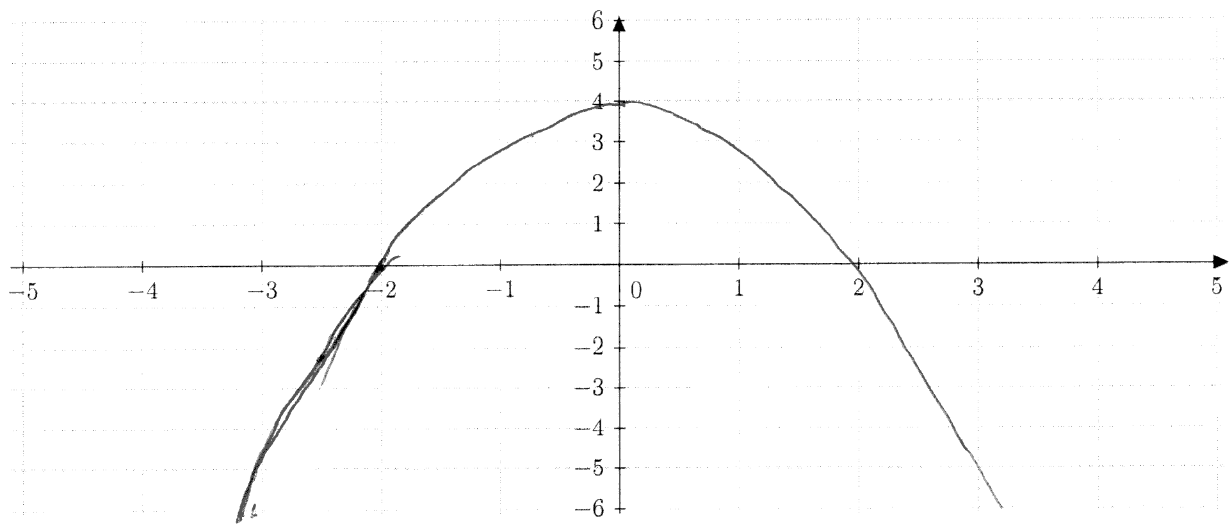
D. $\frac{d}{dx}(\sin(\cos(\tan(x)))) = \cos(\cos(\tan(x)))\cdot\left[\frac{d}{dx}\cos(\tan(x))\right]$

$= \cos(\cos(\tan(x)))\cdot(-\sin(\tan(x)))\left[\frac{d}{dx}\tan(x)\right]$

$= -\cos(\cos(\tan(x)))\cdot\sin(\tan(x))\cdot\sec^2(x)$

9. (8 points) Given the graph of $y = k(x)$, sketch the graph of $y = k'(x)$ in the empty plot below.





10. (8 points) Find the equation of the tangent line to $y = \cos(x)$ at $x = \frac{\pi}{2}$.

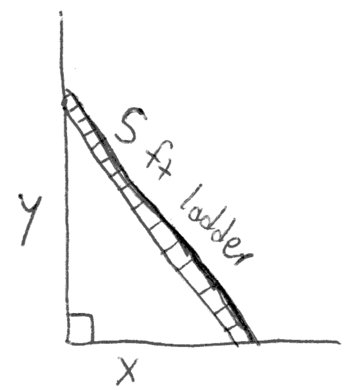
$\frac{dy}{dx} = \frac{d}{dx}\cos(x) = -\sin(x)$ so $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -\sin(\frac{\pi}{2}) = -1$

The tangent line goes through $(x,y) = (\frac{\pi}{2}, \cos(\frac{\pi}{2})) = (\frac{\pi}{2}, 0)$.

$y - 0 = -1(x - \frac{\pi}{2})$

$y = -x + \frac{\pi}{2}$

11. (8 points) A 5-foot ladder rests against the wall. The bottom of the ladder slides away from the wall at a rate of 2 feet/second. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 3 feet from the wall?



Want: $\frac{dy}{dt}$ when $x=3$ ft.

Know: $\frac{dx}{dt}=2$ ft/sec

$$x^2+y^2=5^2$$

$$\frac{d}{dt}\left[x^2+y^2\right]=\frac{d}{dt}5^2$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$2y\frac{dy}{dt}=-2x\frac{dx}{dt}$$

$$\frac{dy}{dt}=-\frac{x}{y}\cdot\frac{dx}{dt}$$

When $x=3$ ft, $3^2+y^2=5^2$, $y^2=5^2-3^2=25-9=16$, and $y=\sqrt{16}=4$ ft. $\frac{dy}{dt}=-\frac{3}{4}\cdot 2=-\frac{3}{2}\ \frac{\text{ft}}{\text{sec}}$