Name Solutions Signature ______

Math 220 – Exam 3 – November 12, 2014

1. (8 points) Use a linearization for the function $f(x) = e^x$ at $x = 0$ to approximate $e^{-.01}$.

$L(x) = f'(0)(x-0) + f(0)$

$f'(x) = e^x$ $\quad f(0) = e^0 = 1$ $\quad f'(0) = e^0 = 1$

$L(x) = 1 \cdot (x-0) + 1 = x+1$

$-.01$ is close to 0 so

$e^{-.01} = f(-.01) \approx L(-.01) = -.01 + 1 = .99$

2. (8 points) Find the absolute minimum and maximum of $m(x) = 2x^3 - 6x + 4$ on the interval [0,2].

$m'(x) = 6x^2 - 6 = 6(x^2-1) = 6(x-1)(x+1)$ is defined everywhere. $m'(x) = 0$ when $x = \pm 1$.

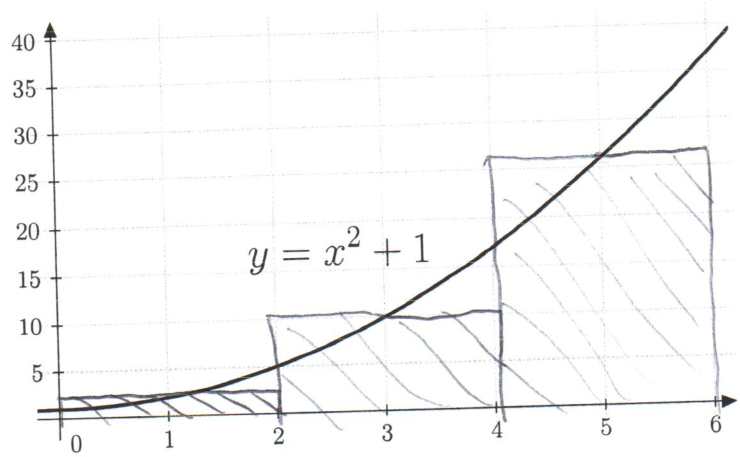
The only critical point in $[0,2]$ is at $x=1$.

$m(0) = 2\cdot(0)^3 - 6\cdot(0) + 4 = 4$

$m(1) = 2\cdot(1)^3 - 6\cdot(1) + 4 = 0$

$m(2) = 2(2)^3 - 6\cdot(2) + 4 = 8$

On $[0,2]$, $m(x)$ has an absolute min at $(1,0)$ and an absolute max at $(2,8)$



3. (8 points) Estimate $\int_0^6 (x^2+1)\,dx$ by using $n = 3$ subintervals, taking the sampling points to be midpoints. In the language of our textbook, this is M_3. Also, illustrate the rectangles on the graph above.

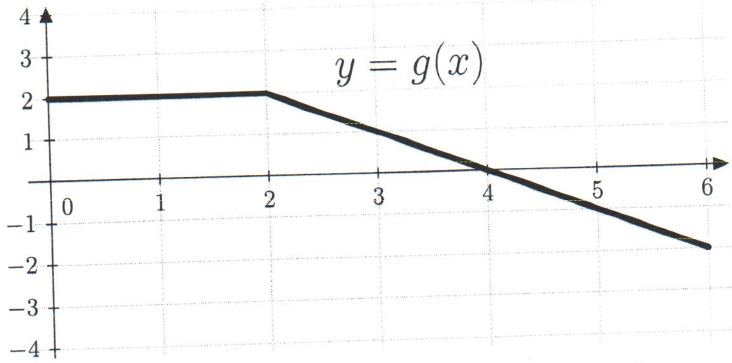
$\Delta x = \frac{6-0}{3} = 2.$ $\qquad x_i = 0 + i\Delta x = 2i$

$$\int_0^6 (x^2+1)\,dx \approx M_3 = (1^2+1)\cdot 2 + (3^2+1)\cdot 2 + (5^2+1)\cdot 2$$
$$= 4 + 20 + 52$$
$$= 76$$

4. (3 points) Find the differential dy if $y = \ln(x)$.

$$\frac{dy}{dx} = \frac{1}{x}$$

$$dy = \frac{1}{x}\cdot dx$$



5. (2 points each) The graph of $y = g(x)$ is shown above. Evaluate the following definite integrals. (You do not need to show your work.)

A. $\int_0^2 g(x)\, dx =$ $2 \cdot 2 = 4$

B. $\int_2^4 g(x)\, dx =$ $\frac{1}{2} \cdot 2 \cdot 2 = 2$

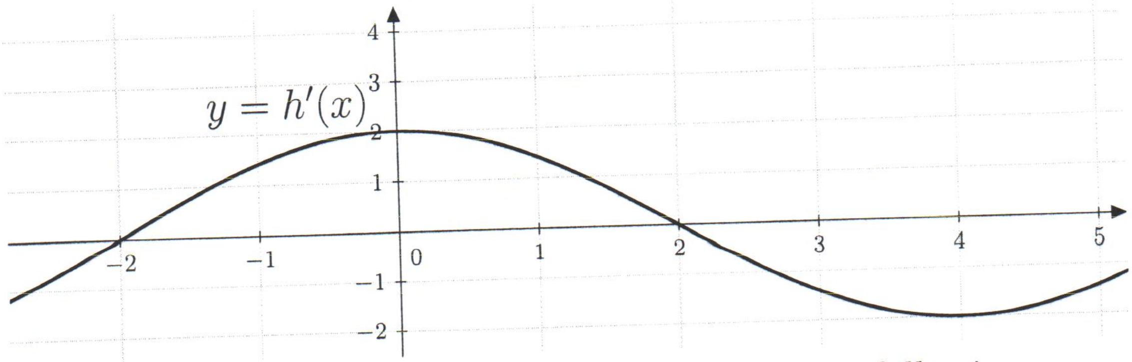
C. $\int_4^6 g(x)\, dx =$ $-\frac{1}{2} \cdot 2 \cdot 2 = -2$

D. $\int_0^6 g(x)\, dx =$ $4 + 2 - 2 = 4$

6. (4 points each) Find the following most general antiderivatives. I hope that you 'C' what I mean.

A. $\int \left(\cos(x) + 4x + \frac{1}{x}\right)\, dx =$ $\sin(x) + 2x^2 + \ln|x| + C$

B. $\int \left(3e^x + 4\sin(x) + 7\sec^2(x)\right)\, dx =$ $3e^x - 4\cos(x) + 7\tan(x) + C$



7. (2 points each) $y = h'(x)$ is plotted above. Find the following:

A. Interval(s) where $h(x)$ is increasing: $(-2, 2)$

B. Interval(s) where $h(x)$ is decreasing: $(-3,-2)$, $(2, 5.25)$

C. x-coordinate(s) where $h(x)$ has a local max: $x=2$

D. x-coordinate(s) where $h(x)$ has a local min: $x=-2$

E. Interval(s) where $h(x)$ is concave up: $(-3, 0)$, $(4, 5.25)$

F. Interval(s) where $h(x)$ is concave down: $(0, 4)$

G. x-coordinate(s) where $h(x)$ has an inflection point: $x=0$ and $x=4$

8. (3 points each) For the function $w(x)$, one has $w''(x) = \dfrac{2(x-1)}{x^2+3}$. Find the following:

A. Interval(s) where $w(x)$ is concave up: $(1, \infty)$

B. Interval(s) where $w(x)$ is concave down: $(-\infty, 1)$

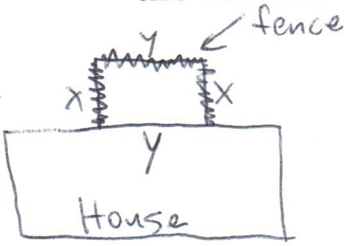
C. x-coordinate(s) where $w(x)$ has an inflection point: $x=1$

$\dfrac{2}{x^2+3} > 0$ for all x

$x-1 > 0$ for $x>1$

$x-1 < 0$ for $x<1$

9. (10 points) A homeowner with 16 feet of fencing wants to enclose a rectangular area against the side of her house. What dimensions will maximize the fenced-in area? (Note that three sides of the rectangle will be formed from fencing, and the house will serve as the fourth side of the rectangle. Make sure to justify why your answer corresponds to the absolute maximum.)



Maximize $A = xy$

$2x + y = 16$ so $y = 16 - 2x$,
Note, x must be in the interval $[0,8]$; otherwise, either x or y is negative.

Maximize $A(x) = x(16-2x) = 16x - 2x^2$ on $[0,8]$.
$A'(x) = 16 - 4x$ is always defined. $0 = A'(x)$ when $4x = 16$, which holds when $x = 4$.

$A(0) = 16(0) - 2(0)^2 = 0$
$A(4) = 16(4) - 2(4)^2 = 64 - 32 = 32$
$A(8) = 16(8) - 2(8)^2 = 128 - 128 = 0$
$\Rightarrow$ so the max occurs when $x = 4$ ft, $y = 16 - 2\cdot 4 = 8$ ft, and $A = 4\cdot 8 = 32$ ft

Alternatives to Closed Interval Method

1st Derivative Test:

$A'(x)$: $+$ | $-$ at 4

2nd Derivative Test:

$A''(x) = -4 < 0$ for all x

10. (8 points) Find $v(x)$ if $v''(x) = 6x + 2$, $v'(0) = 1$, and $v(0) = 2$.

$\int (6x+2)\,dx = 3x^2 + 2x + C$ so $v'(x) = 3x^2 + 2x + C$ for some C.

$1 = v'(0) = 3\cdot(0)^2 + 2\cdot(0) + C = C$ so $v'(x) = 3x^2 + 2x + 1$.

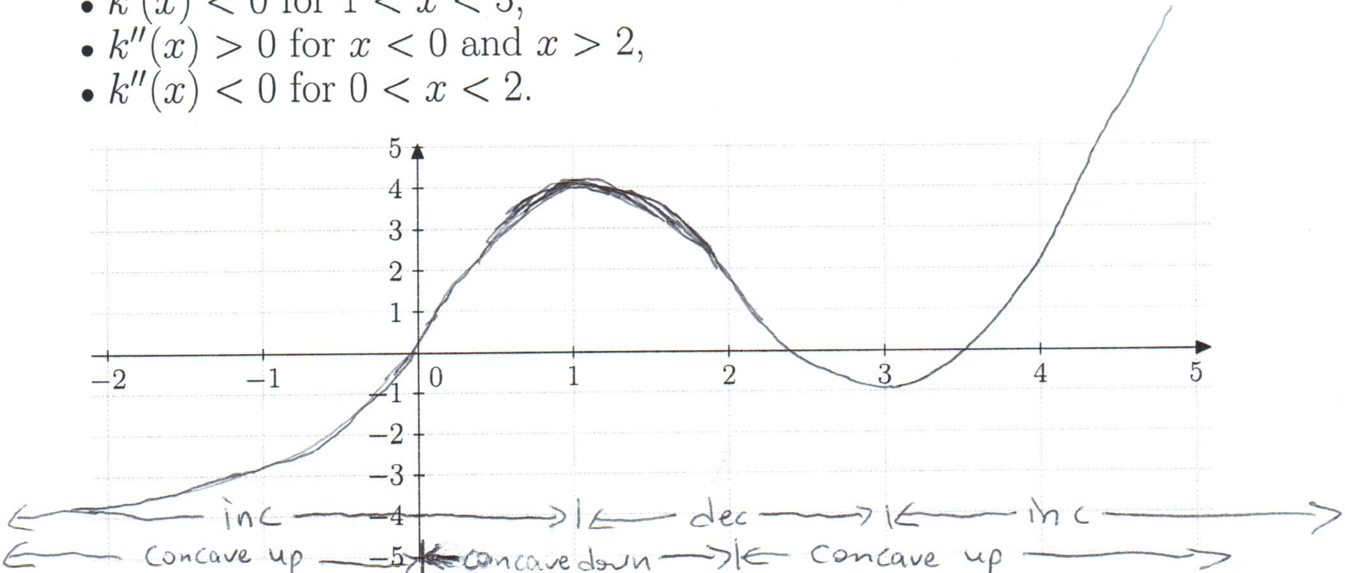
$\int (3x^2 + 2x + 1)\,dx = x^3 + x^2 + x + D$

$v(x) = x^3 + x^2 + x + D$ for some constant D.

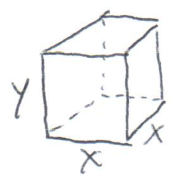
$2 = v(0) = 0^3 + 0^2 + 0 + D = D$ so $v(x) = x^3 + x^2 + x + 2$

11. (6 points) Draw the graph of a function $k(x)$ that satisfies:

- $k'(x) > 0$ for $x < 1$ and $x > 3$,
- $k'(x) < 0$ for $1 < x < 3$,
- $k''(x) > 0$ for $x < 0$ and $x > 2$,
- $k''(x) < 0$ for $0 < x < 2$.



12. (10 points) A rectangular open-topped box is to have a square base and volume 12 ft^3. If material for the base costs \$3 per ft^2 and material for the sides costs \$1 per ft^2, what dimensions minimize the cost of the box? (Justify why your answer is an absolute minimum.)



Minimize Cost $C = 1 \cdot 4xy + 3 \cdot x^2 = 4xy + 3x^2$

Volume $12 = x^2 y$ so $y = \frac{12}{x^2}$.

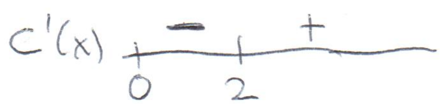
Minimize $C(x) = 4x\left(\frac{12}{x^2}\right) + 3x^2 = \frac{48}{x} + 3x^2$ on $(0, \infty)$.

$C'(x) = -\frac{48}{x^2} + 6x$ is defined on $(0, \infty)$, and

$C'(x) = 0$ when $6x = \frac{48}{x^2} \iff 6x^3 = 48 \iff x^3 = 8$

$\iff x = 2$.

Option 1: 1st Derivative Test



Option 2: 2nd Derivative Test

$C''(x) = \frac{96}{x^3} + 6 > 0$ on $(0, \infty)$

Cost is minimized when $x = 2$ ft, $y = \frac{12}{2^2} = 3$ ft, and

$C(x) = \frac{48}{2} + 3 \cdot 2^2 = 24 + 12 = \$36$