Name ______________________________ Signature ______________________________

Math 220 – Final Exam – December 17, 2014

1. (6 points) Let $g(x) = x^2 + 1$. **Using the limit definition of the derivative,** find $g'(3)$.

2. (6 points) Using a linearization for $f(x) = \sqrt{x}$ at $x = 1$, estimate $\sqrt{1.02}$.

3. (3 points) At time t minutes, Alice has a velocity of $v_A(t)$ ft/min, and Fred has a velocity of $v_F(t)$ ft/min. Describe the meaning of the integral $\displaystyle\int_2^5 (v_A(t) - v_F(t))\, dt$.

4. (6 points) Find the area bounded between $y = 2x$ and $y = x^2$.

5. (6 points) Find the volume of the solid formed by rotating the region bounded by $y = 0$, $x = 1$, and $y = x^3$ around the x-axis.

6. (6 points) Find $m(x)$ provided that $m''(x) = e^x$, $m'(0) = 2$, and $m(0) = 5$.

7. (4 points each) Compute the following:

A. $\dfrac{d}{dx} \displaystyle\int_3^x t^5 \cos(t^3 + 42t)\, dt$

B. $\dfrac{d}{dx} \left(\dfrac{e^{x^2}}{2x^5 + x} \right)$

C. $\dfrac{d}{dx} \left(\sin(x) \cdot \arctan(x)\right)$

D. $\displaystyle\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

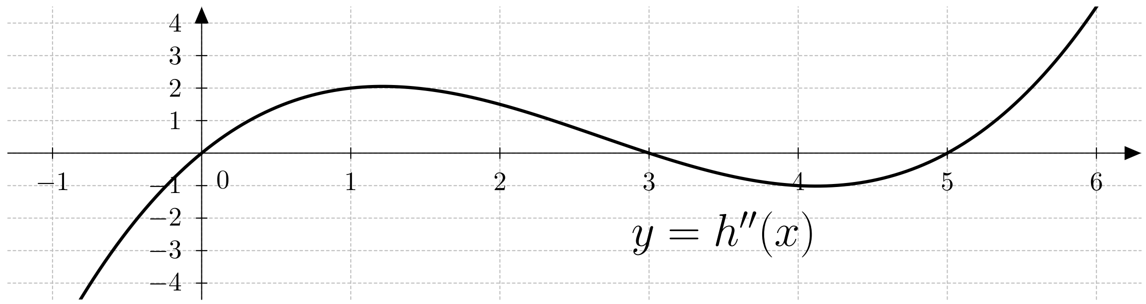
E. $\displaystyle\lim_{x \to \infty} \frac{7x^5 + 3x^2 + 3}{6x^5 + 4x^4 + 2x}$

8. (6 points each) Compute the following:

A. $\dfrac{dy}{dx}$ if $y^4 + xy = x^3 - x + 2$

B. $\displaystyle\int x \cdot \sec^2(x^2)\, dx$

C. $\displaystyle\int_0^{\pi/2} \sqrt{\sin(t)} \cos(t)\, dt$



9. (2 points each) Above is a graph of $y = h''(x)$. Find:

A. Interval(s) where $h(x)$ is concave up: ____________________

B. Interval(s) where $h(x)$ is concave down: ____________________

C. x-coordinate(s) where $h(x)$ has an inflection point: ______________

10. (3 points) Let $r(t)$ denote the rate in gallons per hour at which water is flowing into a tank t hours after noon. Describe the meaning of $\int_2^4 r(t)\,dt$.

11. (6 points) Find the absolute maximum and minimum of $w(x) = x^3 + 3x^2 - 9x + 1$ on $[0, 2]$.

12. (7 points) At noon, Mary is a mile north of Sally. Mary is hiking north at a rate of 1 mile per hour, and Sally is walking east at a rate of 2 miles per hour. How fast is the distance between Mary and Sally changing at 2:00 PM?

13. (7 points) 6 ft^2 of material is available to make a rectangular box with an open top. The length of its base must be twice the width. Find the dimensions that maximize the volume of the box. (Justify why your answer is an absolute maximum.)