Name Solutions Signature ____________

**Math 220 – Exam 1 – February 13, 2014**

**1.** (5 points) Write an equation for the line with slope 2 that passes through the point $(0, 1)$.

$y = mx + b$

slope is 2 $\Rightarrow m = 2$

passes through $(0,1)$ $\Rightarrow$ y-int is at 1 $\Rightarrow b = 1$

$\boxed{y = 2x + 1}$

**2.** (4 points) If $r(x) = x + 5$ and $u(x) = x^3$, find $r(u(x))$.

$r(u(x)) = r(x^3) = \boxed{x^3 + 5}$

**3.** (9 points) Find the constant $c$ that makes the following function continuous.

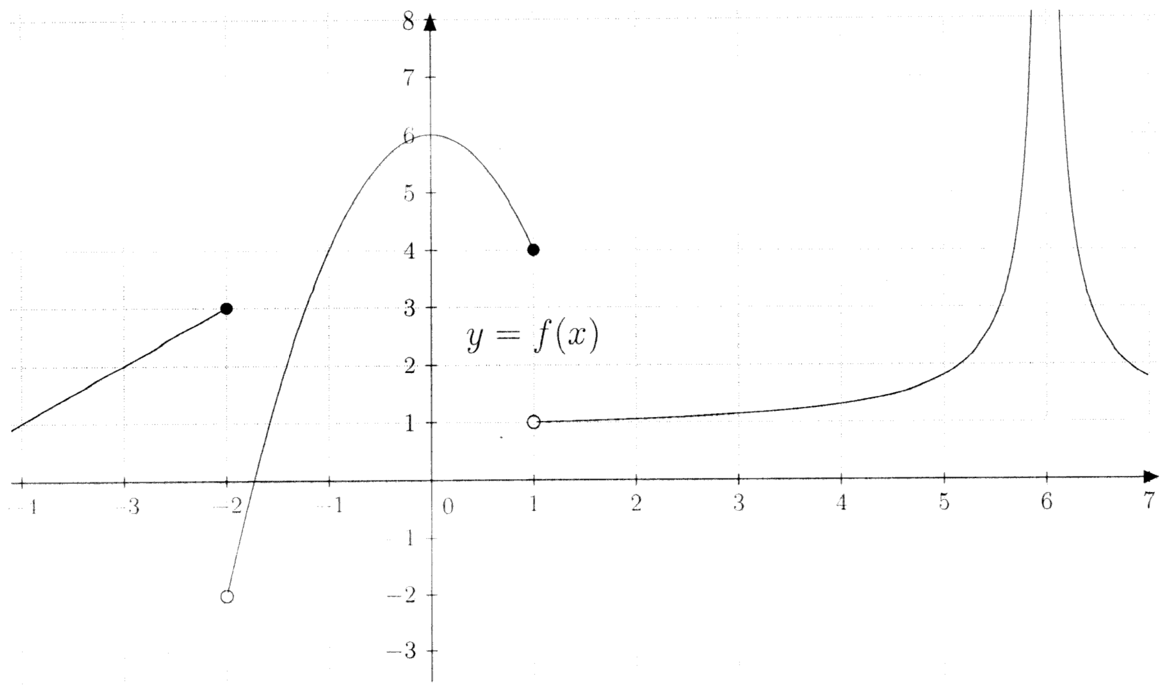
$$q(x) = \begin{cases} 3 & \text{if } x > 2 \\ x + c & \text{if } x \leq 2 \end{cases}$$

$\lim_{x \to 2^+} q(x) = \lim_{x \to 2^+} 3 = 3$

$\lim_{x \to 2^-} q(x) = \lim_{x \to 2^-} (x + c) = 2 + c.$

We need $\lim_{x \to 2^+} q(x) = \lim_{x \to 2^-} q(x)$ so $3 = 2 + c.$

Hence, $\boxed{c = 1.}$




**4.** (4 points each) Consider the graph of $y = f(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

**A.** $\lim_{x\to 0} f(x)$ = 6

**E.** $\lim_{x\to 1^-} f(x)$ = 4

**B.** $\lim_{x\to -2^-} f(x)$ = 3

**F.** $\lim_{x\to 1^+} f(x)$ = 1

**C.** $\lim_{x\to -2^+} f(x)$ = −2

**G.** $\lim_{x\to 1} f(x)$ does not exist

**D.** $\lim_{x\to 6} f(x)$ = +∞

**H.** $f(1)$ = 4

**5.** (7 points each) Evaluate the following limits.

**A.** $\lim_{x\to 0} \frac{3\sin(x)}{x}$ $= 3\left[\lim_{x\to 0} \frac{\sin(x)}{x}\right] = 3\cdot 1 = 3$

**B.** $\lim_{x\to 5} \frac{x-5}{x^2-25}$ $= \lim_{x\to 5} \frac{x-5}{(x-5)(x+5)} = \lim_{x\to 5} \frac{1}{x+5} = \frac{1}{5+5} = \frac{1}{10}$

**C.** $\lim_{x\to 4} \frac{2-\sqrt{x}}{x-4}$ $= \lim_{x\to 4} \frac{2-\sqrt{x}}{x-4} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} = \lim_{x\to 4} \frac{4-x}{(x-4)(2+\sqrt{x})}$

$= \lim_{x\to 4} \frac{-(x-4)}{(x-4)(2+\sqrt{x})} = \lim_{x\to 4} \frac{-1}{2+\sqrt{x}} = \frac{-1}{2+\sqrt{4}} = \frac{-1}{4}$

**D.** $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$

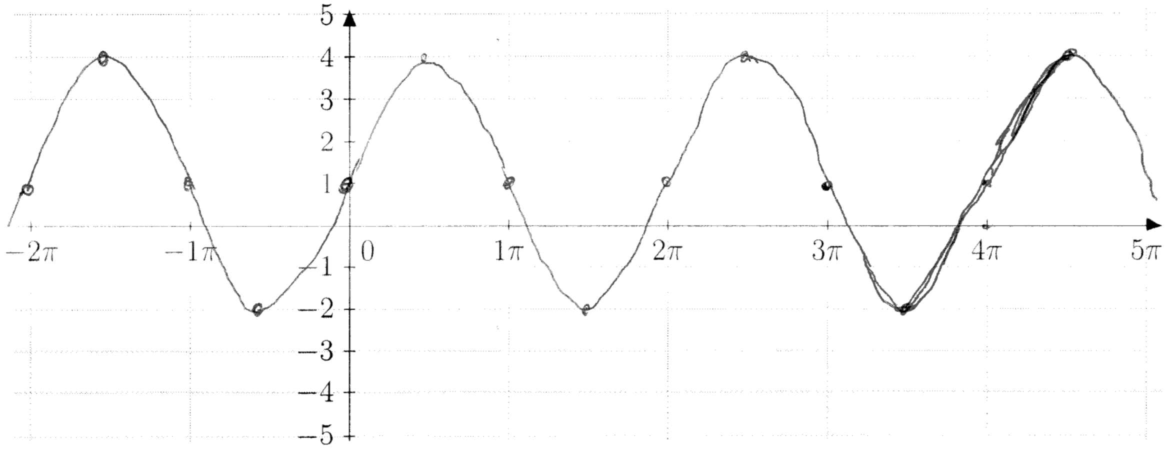
$-1 \le \sin\left(\frac{1}{x}\right) \le 1 \quad (x\neq 0)$

Since $x^2 \ge 0$, $-x^2 \le x^2\sin\left(\frac{1}{x}\right) \le x^2 \quad (x\neq 0)$.

$\lim_{x\to 0} (-x^2) = -0^2 = 0$. $\lim_{x\to 0} x^2 = 0^2 = 0$.

By the Squeeze Theorem, $\lim_{x\to 0} x^2\sin\left(\frac{1}{x}\right) = 0$.

**6.** (8 points) Sketch the graph of $y = 3\sin(x) + 1$.




**7.** (5 points) Given that $\lim_{x\to 2} w(x) = 3$ and $\lim_{x\to 2} h(x) = 5$, find $\lim_{x\to 2} \dfrac{w(x)+1}{h(x)}$.

$$\lim_{x\to2} \frac{w(x)+1}{h(x)} = \frac{\lim_{x\to2} (w(x)+1)}{\lim_{x\to2} h(x)} = \frac{\left(\lim_{x\to2} w(x)\right) + \left(\lim_{x\to2} 1\right)}{\lim_{x\to2} h(x)} = \frac{3+1}{5} = \frac{4}{5}$$

**8.** (9 points) Suppose that a particle has position function $s(t) = t^2 + 1$ meters at time $t$ seconds. Find the average velocity over the time interval $[2, 4]$.

$$\frac{s(4)-s(2)}{4-2} = \frac{(4^2+1)-(2^2+1)}{2} = \frac{17-5}{2} = \frac{12}{2} = 6 \text{ m/s}$$