Name Solutions Signature ______________________

Math 220 – Exam 2 (A) – March 13, 2014

1. (10 points) Find $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{16x^2+3x+2}}{5x+9}$.

(Ver. A) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{16x^2+3x+2}}{5x+9}=\lim_{x\to-\infty}\frac{\frac{\sqrt{16x^2+3x+2}}{x}}{\frac{5x+9}{x}}=\lim_{x\to-\infty}\frac{\frac{\sqrt{16x^2+3x+2}}{-\sqrt{x^2}}}{5+\frac{9}{x}}=\lim_{x\to-\infty}\frac{-\sqrt{16+\frac{3}{x}+\frac{2}{x^2}}}{5+\frac{9}{x}}=\frac{-\sqrt{16+0+0}}{5+0}=-\frac{4}{5}$

(Ver. B) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{16x^2+5x+2}}{5x+4}=\lim_{x\to-\infty}\frac{\frac{\sqrt{16x^2+5x+2}}{x}}{\frac{5x+4}{x}}=\lim_{x\to-\infty}\frac{\frac{\sqrt{16x^2+5x+2}}{-\sqrt{x^2}}}{5+\frac{4}{x}}=\lim_{x\to-\infty}\frac{-\sqrt{16+\frac{5}{x}+\frac{2}{x^2}}}{5+\frac{4}{x}}=\frac{-\sqrt{16+0+0}}{5+0}=-\frac{4}{5}$

(Ver. C) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{16x^2+4x+6}}{5x+7}=\lim_{x\to-\infty}\frac{\frac{\sqrt{16x^2+4x+6}}{x}}{\frac{5x+7}{x}}=\lim_{x\to-\infty}\frac{\frac{\sqrt{16x^2+4x+6}}{-\sqrt{x^2}}}{5+\frac{7}{x}}=\lim_{x\to-\infty}\frac{-\sqrt{16+\frac{4}{x}+\frac{6}{x^2}}}{5+\frac{7}{x}}=\frac{-\sqrt{16+0+0}}{5+0}=-\frac{4}{5}$

(Ver. B $g(x)=x^2+3$) (Ver. C $g(x)=x^2+5$)

2. (10 points) Let $g(x)=x^2+8$. Using the **limit definition of the derivative**, find $g'(3)$.

(Ver. A) $\displaystyle g'(3)=\lim_{h\to0}\frac{g(3+h)-g(3)}{h}=\lim_{h\to0}\frac{((3+h)^2+8)-(3^2+8)}{h}=\lim_{h\to0}\frac{9+6h+h^2+8-9-8}{h}=\lim_{h\to0}\frac{6h+h^2}{h}$

$\displaystyle=\lim_{h\to0}(6+h)=6$

(Ver. B) $\displaystyle g'(3)=\lim_{h\to0}\frac{g(3+h)-g(3)}{h}=\lim_{h\to0}\frac{((3+h)^2+3)-(3^2+3)}{h}=\lim_{h\to0}\frac{9+6h+h^2+3-9-3}{h}$

$\displaystyle=\lim_{h\to0}\frac{6h+h^2}{h}=\lim_{h\to0}(6+h)=6$

(Ver. C) $\displaystyle g'(3)=\lim_{h\to0}\frac{g(3+h)-g(3)}{h}=\lim_{h\to0}\frac{((3+h)^2+5)-(3^2+5)}{h}=\lim_{h\to0}\frac{9+6h+h^2+5-9-5}{h}$

$\displaystyle=\lim_{h\to0}\frac{6h+h^2}{h}=\lim_{h\to0}(6+h)=6$

3. (6 points) Suppose that a waiter brings you a cup of hot coffee. Let $F(t)$ denote the temperature in degrees Fahrenheit of the coffee after t minutes. Is $F'(3)$ positive or negative? Explain your answer.

$F'(3)$ is negative because the coffee's temperature is decreasing three minutes after the waiter brings it.

(Ver. B $s(t)=t^2+2$) (Ver. C $s(t)=t^2+3$)

4. (9 points) Suppose that the position of a particle is given by $s(t) = t^2 + 4$ meters at time t seconds. Find the instantaneous velocity at time $t = 3$ seconds.

$s'(t) = 2t$

$s'(3) = 2 \cdot 3 = 6$ m/sec

(Ver. B $y=x^2+2$) (Ver. C $y=x^2+1$)

5. (10 points) Find the tangent line to $y = x^2 + 3$ at $x = 2$.

(Ver A) The line goes through $(2, 2^2+3) = (2,7)$ and has slope $\frac{d}{dx}(x^2+3)\Big|_{x=2} = 2x\Big|_{x=2} = 4$. $\boxed{y-7 = 4(x-2)}$

(Ver B) The line goes through $(2, 2^2+2) = (2,6)$ and has slope $\frac{d}{dx}(x^2+2)\Big|_{x=2} = 2x\Big|_{x=2} = 4$. $\boxed{y-6 = 4(x-2)}$

(Ver C) The line goes through $(2, 2^2+1) = (2,5)$ and has slope $\frac{d}{dx}(x^2+1)\Big|_{x=2} = 2x\Big|_{x=2} = 4$ $\boxed{y-5 = 4(x-2)}$

6. (10 points) Let $g(x) = x^x$. Find $g'(x)$.

$$\ln(g(x)) = \ln(x^x) = x\ln(x)$$

$$\frac{d}{dx}\ln(g(x)) = \frac{d}{dx}\left[x\ln(x)\right]$$

$$\frac{g'(x)}{g(x)} = 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x)+1$$

$$g'(x) = (\ln(x)+1)g(x) = (\ln(x)+1)x^x.$$

(Ver. B $x^2+3y^2=3$) (Ver C $x^2+3y^2=3$)

7. (10 points) Find $\frac{dy}{dx}$ for $x^2+4y^2=3$.

(Ver A)

$$\frac{d}{dx}\left[x^2+4y^2\right] = \frac{d}{dx}3$$

$$2x + 8y\frac{dy}{dx} = 0$$

$$8y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{8y}$$

$$\frac{dy}{dx} = \frac{-x}{4y}$$

(Ver C)

$$\frac{d}{dx}\left[x^2+5y^2\right] = \frac{d}{dx}7$$

$$2x+10y\frac{dy}{dx} = 0$$

$$10y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{10y}$$

$$\frac{dy}{dx} = \frac{-x}{5y}$$

(ver B)

$$\frac{d}{dx}\left[x^2+3y^2\right] = \frac{d}{dx}3$$

$$2x+6y\frac{dy}{dx} = 0$$

$$6y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{6y}$$

$$\frac{dy}{dx} = \frac{-x}{3y}$$

8. (7 points each) Find the following derivatives. You do not need to simplify.

A. $\frac{d}{dx}\left(\arctan(x)+\sqrt{x}\right) = \frac{1}{1+x^2} + \frac{1}{2}x^{-1/2}$

B. $\frac{d}{dx}\left(e^{5x^3+2x}\right) = e^{5x^3+2x}(15x^2+2)$

(Ver. B) $\frac{d}{dx}\left(e^{4x^3+5x}\right) = e^{4x^3+5x}(12x^2+5)$

(Ver. C) $\frac{d}{dx}\left(e^{4x^3+3x}\right) = e^{4x^3+3x}(12x^2+3)$

C. $\frac{d}{dx}\left(\frac{3x^2+2}{x^8+x^4}\right) = \frac{(6x)(x^8+x^4)-(3x^2+2)(8x^7+4x^3)}{(x^8+x^4)^2}$

(Ver. B) $\frac{d}{dx}\left(\frac{2x^3+5}{x^7+x^6}\right) = \frac{(6x)(x^7+x^6)-(2x^3+5)(7x^6+6x^5)}{(x^7+x^6)^2}$

(Ver C) $\frac{d}{dx}\left(\frac{3x^2+3}{x^9+x^2}\right) = \frac{(6x)(x^9+x^2)-(3x^2+3)(9x^8+2x)}{(x^9+x^2)^2}$

D. $\frac{d}{dx}\left(3^x\cdot\cos(x)\right) = 3^x\ln(3)\cos(x) + 3^x\cdot(-\sin(x))$

(Ver B) $\frac{d}{dx}\left(5^x\cos(x)\right) = 5^x\ln(5)\cos(x) + 5^x\cdot(-\sin(x))$

(Ver C) $\frac{d}{dx}\left(7^x\cos(x)\right) = 7^x\ln(7)\cos(x) + 7^x\cdot(-\sin(x))$

E. $\frac{d}{dx}\left(\ln(\sin(x^2+1))\right) = \frac{1}{\sin(x^2+1)}\cdot\cos(x^2+1)\cdot 2x$

(Ver B) $\frac{d}{dx}\ln(\sin(x^3+1)) = \frac{1}{\sin(x^3+1)}\cdot\cos(x^3+1)\cdot 3x^2$

(Ver C) $\frac{d}{dx}\ln(\sin(x^4+1)) = \frac{1}{\sin(x^4+1)}\cdot\cos(x^4+1)\cdot 4x^3$