Name Solutions Signature

Math 220 – Exam 3 (Version A) – April 17, 2014

1. (12 points) Find the absolute minimum and maximum of $m(x) = x^3 - 3x + 2$ on the interval [0,2].

$m'(x) = 3x^2 - 3 = 3(x^2-1) = 3(x-1)(x+1)$ is defined for all x.

$0 = m'(x) = 3(x-1)(x+1)$ when $x = \pm 1$.

1 is the only critical point in $[0,2]$

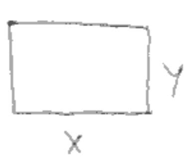
$m(0) = 0^3 - 3\cdot 0 + 2 = 2$

$m(1) = 1^3 - 3\cdot 1 + 2 = 0$

$m(2) = 2^3 - 3\cdot 2 + 2 = 4$

The absolute min is $(1,0)$, and the absolute max is $(2,4)$.

2. (12 points) What is the smallest perimeter possible for a rectangle of area 4 ft^2? (Explain why your answer corresponds to a minimum.)



Minimize perimeter $p = 2x + 2y$

Area is $4 = xy$ so $y = \frac{4}{x}$.

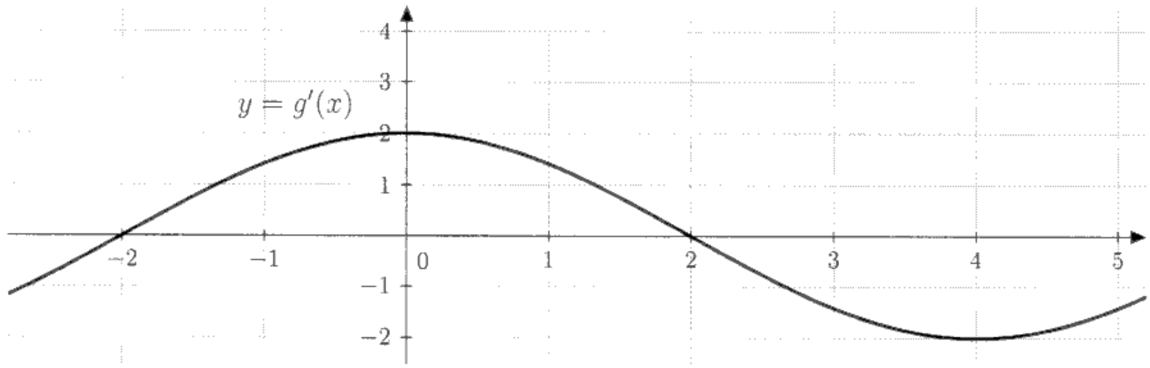
Minimize $p(x) = 2x + 2\left(\frac{4}{x}\right) = 2x + \frac{8}{x}$ on $(0,\infty)$.

$p'(x) = 2 - \frac{8}{x^2}$ is defined for all x in $(0,\infty)$.

$0 = p'(x) = 2 - \frac{8}{x^2} \Leftrightarrow \frac{8}{x^2} = 2 \Leftrightarrow 8 = 2x^2 \Leftrightarrow 4 = x^2 \Leftrightarrow x = \pm 2$.

$x = 2$ is the only critical point on $(0,\infty)$.

$p''(x) = \frac{16}{x^3} > 0$ on $(0,\infty)$. Hence, $p(x)$ obtains its absolute minimum when $x = 2$, $y = \frac{4}{2} = 2$, and $p(2) = 2\cdot 2 + \frac{8}{2} = 8$.



3. (2 points each) $y = g'(x)$ is plotted above. Find the following:

A. Interval(s) where $g(x)$ is increasing: $(-2, 2)$

B. Interval(s) where $g(x)$ is decreasing: $(-\infty, -2), (2, \infty)$ or $(-2.9, -2), (2, 5.2)$

C. x-coordinate(s) where $g(x)$ has a local max: $x = 2$

D. x-coordinate(s) where $g(x)$ has a local min: $x = -2$

E. Interval(s) where $g(x)$ is concave up: $(-\infty, 0), (4, \infty)$ or $(-2.9, 0), (4, 5.2)$

F. Interval(s) where $g(x)$ is concave down: $(0, 4)$

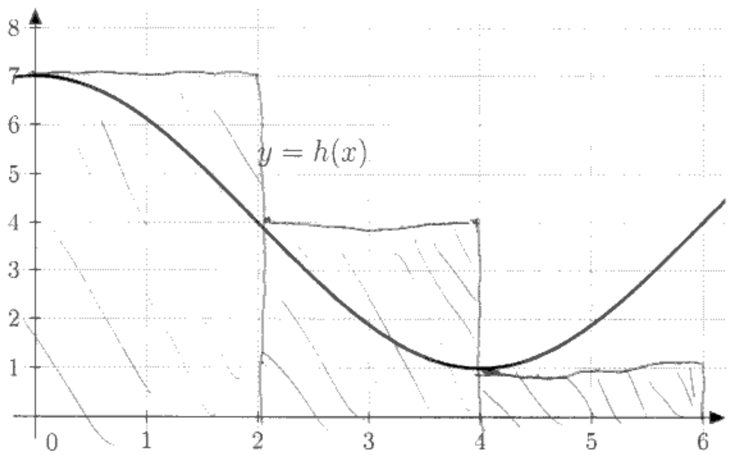
G. x-coordinate(s) where $g(x)$ has an inflection point: $x = 0,\ x = 4$

4. (3 points each) For the function $w(x)$, one has $w''(x) = \dfrac{3(x-2)}{x^2+1}$. Find the following:

A. Interval(s) where $w(x)$ is concave up: $(2, \infty)$

B. Interval(s) where $w(x)$ is concave down: $(-\infty, 2)$

C. x-coordinate(s) where $w(x)$ has an inflection point: $x = 2$



5. (9 points) Estimate the area below $y = h(x)$ and above the x-axis for $0 \leq x \leq 6$ by using $n = 3$ subintervals, taking the sampling points to be left endpoints. In the language of our textbook, this is L_3. Also, illustrate the rectangles on the graph above.

$$\Delta x = \frac{6-0}{3} = 2$$

$$L_3 = h(0)\cdot 2 + h(2)\cdot 2 + h(4)\cdot 2$$

$$= 7\cdot 2 + 4\cdot 2 + 1\cdot 2$$

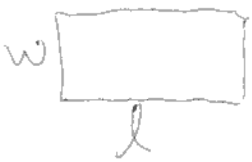
$$= 24$$

6. (6 points each) Find the following most general antiderivatives. I hope that you "C" what I mean.

A. $\int (7 + 2x + 3e^x)\, dx =$ $7x + x^2 + 3e^x + C$

B. $\int (\sec^2(\theta) + \cos(\theta))\, d\theta =$ $\tan(\theta) + \sin(\theta) + C$

7. (12 points) The length of a rectangle is increasing at a rate of 2 ft/s, and its width is increasing at a rate of 5 ft/s. At what rate is the area of the rectangle increasing when the length is 4 ft and the width is 6 ft?



Area $A = lw$. $\frac{dA}{dt} = \frac{dl}{dt} \cdot w + l \cdot \frac{dw}{dt}$

We are given $\frac{dl}{dt} = 2 \frac{ft}{sec}$ and $\frac{dw}{dt} = 5 \frac{ft}{sec}$.

When $l = 4ft$ and $w = 6ft$, we have

$$\frac{dA}{dt} = 2 \cdot 6 + 4 \cdot 5 = 32 \frac{ft^2}{sec}.$$

8. (10 points) Use a linearization for the function $f(x) = \sqrt{x}$ at $x = 4$ to approximate $\sqrt{4.04}$.

$f(x) = \sqrt{x}$ so $f'(x) = \frac{1}{2\sqrt{x}}$.

$L(x) = f(4) + f'(4)(x-4) = \sqrt{4} + \frac{1}{2\sqrt{4}}(x-4) = 2 + \frac{1}{4}(x-4)$.

is the linearization of $f(x)$ at $x=4$.

4.04 is close to 4 so

$$\sqrt{4.04} = f(4.04) \approx L(4.04) = 2 + \frac{1}{4}(4.04 - 4)$$
$$= 2 + \frac{1}{4}(.04) = 2.01$$

9. (10 points) Find the function $k(x)$ provided that $k'(x) = 2x^3 + 3x + 2$ and $k(0) = 2$.

$$\int (2x^3 + 3x + 2)\,dx = \frac{1}{2}x^4 + \frac{3}{2}x^2 + 2x + C$$

So $k(x) = \frac{1}{2}x^4 + \frac{3}{2}x^2 + 2x + C$ for some constant C.

$2 = k(0) = \frac{1}{2}(0)^4 + \frac{3}{2}(0)^2 + 2(0) + C = C$.

Hence, $k(x) = \frac{1}{2}x^4 + \frac{3}{2}x^2 + 2x + 2$