Name Solutions Rec. Instr.

Signature Rec. Time

Math 220

Exam 1

February 5, 2015

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

SHOW YOUR WORK!

Problem	Points	Points Possible	Problem	Points	Points Possible
1		12	7		8
2		12	8		5
3		16	9		8
4		7	10		8
5		6	11		10
6		8	Total Score		100

1. (4 points each) Evaluate the following limits.

A. $\lim_{\theta\to 0}(\tan(\theta)+\theta+1) = \tan(0)+0+1 = 0+0+1 = 1$

B. $\lim_{\theta\to 0}\frac{3\sin(\theta)}{7\theta} = \frac{3}{7}\left(\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}\right) = \frac{3}{7}\cdot 1 = \frac{3}{7}$

C. $\lim_{x\to\infty}\frac{4-3x+9x^2}{3x^2+8x+6} = \frac{9}{3}\left(\lim_{x\to\infty} x^{2-2}\right) = 3\cdot\left(\lim_{x\to\infty} 1\right) = 3\cdot 1 = 3$

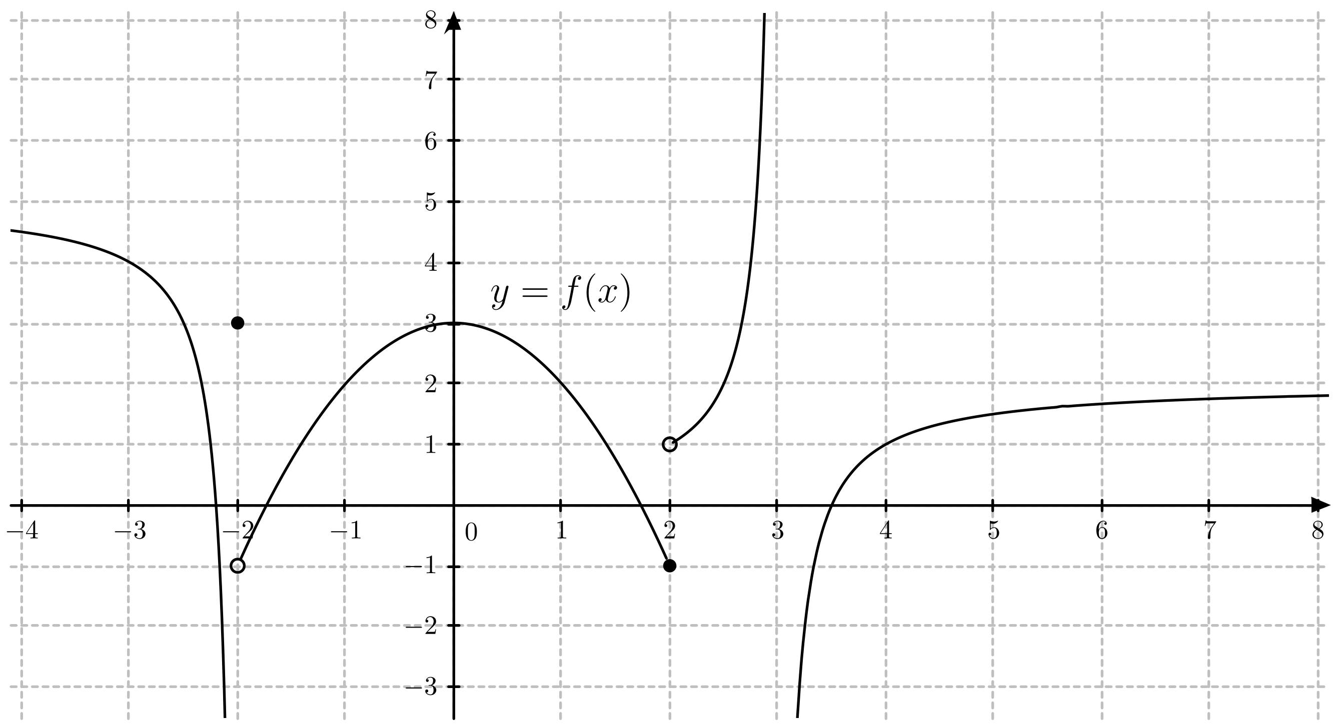
2. (6 points each) Evaluate the following limits.

A. $\lim_{x\to 5}\frac{x^2-4x-5}{x-5} = \lim_{x\to 5}\frac{(x-5)(x+1)}{x-5} = \lim_{x\to 5}(x+1) = 5+1 = 6$

B. $\lim_{t\to 9}\frac{3-\sqrt{t}}{9-t} = \lim_{t\to 9}\left(\frac{3-\sqrt{t}}{9-t}\cdot\frac{3+\sqrt{t}}{3+\sqrt{t}}\right) = \lim_{t\to 9}\frac{9+\cancel{3\sqrt{t}}-\cancel{3\sqrt{t}}-t}{(9-t)(3+\sqrt{t})}$

$= \lim_{t\to 9}\frac{9-t}{(9-t)(3+\sqrt{t})} = \lim_{t\to 9}\frac{1}{3+\sqrt{t}}$

$= \frac{1}{3+\sqrt{9}} = \frac{1}{3+3} = \frac{1}{6}.$



3. (2 points each) Consider the graph of $y = f(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x \to 4} f(x) = 1$

E. $\lim_{x \to 2^-} f(x) = -1$

B. $\lim_{x \to -2} f(x)$ does not exist

F. $\lim_{x \to 2^+} f(x) = 1$

C. $\lim_{x \to \infty} f(x) = 2$

G. $\lim_{x \to 2} f(x)$ does not exist

D. $\lim_{x \to 3^+} f(x) = -\infty$

H. $f(2) = -1$

4. (7 points each) Provided that $4x - 1 \le h(x) \le 2x^2 + 1$ for all x, find $\lim_{x\to 1} h(x)$. (Justify your reasoning, and state the name of any theorem used.)

$$\lim_{x\to 1} (4x-1) = 4\cdot 1 - 1 = 3$$

$$\lim_{x\to 1} (2x^2+1) = 2\cdot 1^2 + 1 = 3$$

By the Squeeze Theorem, $\lim_{x\to 1} h(x) = 3$.

5. (6 points) The position of a particle at time t seconds is given by $s(t) = t^2 - t$ meters. Find the average velocity over the time interval $[2, 3]$ seconds. (Your final answer should include units.)

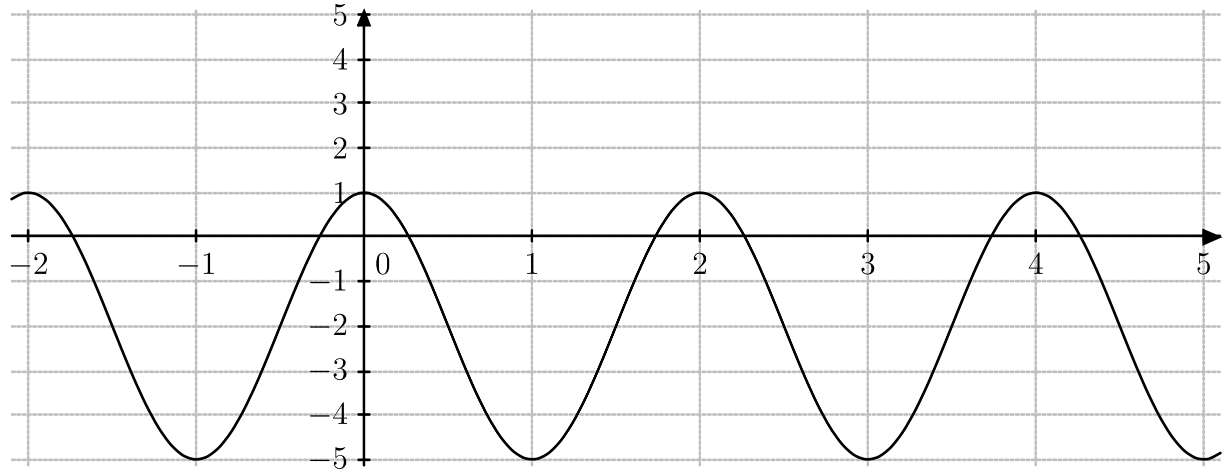
$$\frac{\Delta s}{\Delta t} = \frac{s(3)-s(2)}{3-2} = \frac{(3^2-3)-(2^2-2)}{1} = 6-2 = 4 \text{ m/s}$$

6. (4 points each) Given that $\lim_{x\to -5} m(x) = 4$ and $\lim_{x\to -5} u(x) = 3$, find the following limits.

A. $\lim_{x\to -5} (m(x) + 3u(x)) = 4 + 3\cdot 3 = 4 + 9 = 13$

B. $\lim_{x\to -5} \dfrac{u(x)}{\sqrt{m(x)}} = \dfrac{3}{\sqrt{4}} = \dfrac{3}{2}$

7. (8 points) Sketch the graph of $y = 3\cos(\pi x) - 2$.

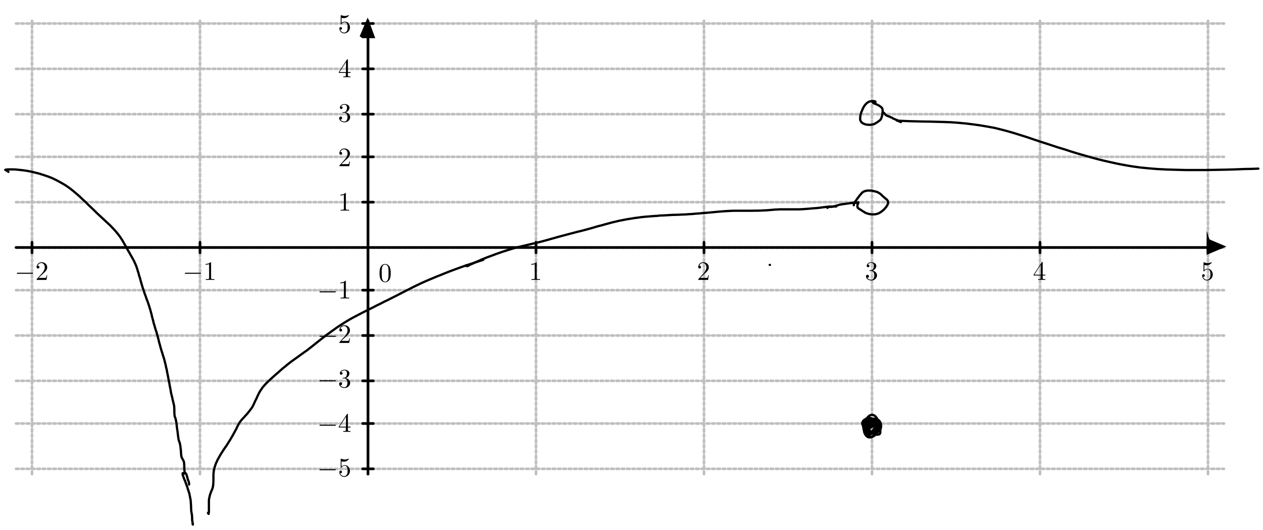


8. (5 points) Write an equation for the line with slope 3 that goes through the point (2,4).

$y - 4 = 3(x-2)$ or

$y = 3x + b$
$4 = 3 \cdot 2 + b$
$4 = 6 + b$
$-2 = b$
$y = 3x - 2$

9. (8 points) Sketch the graph of a function $v(x)$ that satisfies $\lim_{x\to -1} v(x) = -\infty$, $\lim_{x\to 3^-} v(x) = 1$, $\lim_{x\to 3^+} v(x) = 3$, and $v(3) = -4$.



10. (8 points) Use the Intermediate Value Theorem to show that there is a root of $3x^7 - 2^x = 0$ in the interval $(0, 1)$. (Make sure to mention any properties required to apply the Intermediate Value Theorem.)

Let $f(x) = 3x^7 - 2^x$. $f(x)$ is continuous everywhere.

$f(0) = 3 \cdot (0)^7 - 2^0 = 0 - 1 = -1$

$f(1) = 3 \cdot (1)^7 - 2^1 = 3 - 2 = 1$

By the Intermediate Value Theorem, there exists a c in $(0,1)$ with $0 = f(c) = 3c^7 - 2^c$.

11. (10 points) Find the horizontal asymptote(s) for $y = \dfrac{\sqrt{9x^2+5}}{2x-7}$. (Show your work using limits.)

$$\lim_{x\to\infty} \frac{\sqrt{9x^2+5}}{2x-7} = \lim_{x\to\infty} \frac{\frac{\sqrt{9x^2+5}}{x}}{\frac{2x-7}{x}} \underset{\boxed{\substack{x>0 \\ \text{so } x=\sqrt{x^2}}}}{=} \lim_{x\to\infty} \frac{\frac{\sqrt{9x^2+5}}{\sqrt{x^2}}}{2-\frac{7}{x}} = \lim_{x\to\infty} \frac{\sqrt{\frac{9x^2+5}{x^2}}}{2-\frac{7}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{9+\frac{5}{x^2}}}{2-\frac{7}{x}} = \frac{\sqrt{9+0}}{2-0} = \frac{3}{2}$$

$$\lim_{x\to-\infty} \frac{\sqrt{9x^2+5}}{2x-7} = \lim_{x\to-\infty} \frac{\frac{\sqrt{9x^2+5}}{x}}{\frac{2x-7}{x}} \underset{\boxed{\substack{x<0 \text{ so} \\ x=-\sqrt{x^2}}}}{=} \lim_{x\to-\infty} \frac{\frac{\sqrt{9x^2+5}}{-\sqrt{x^2}}}{2-\frac{7}{x}} = \lim_{x\to-\infty} \frac{-\sqrt{\frac{9x^2+5}{x^2}}}{2-\frac{7}{x}}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{9+\frac{5}{x^2}}}{2-\frac{7}{x}} = \frac{-\sqrt{9+0}}{2-0} = -\frac{3}{2}$$

The horizontal asymptotes are $y = \frac{3}{2}$ and $y = -\frac{3}{2}$.