Name Solutions Rec. Instr.
Signature Rec. Time

Math 220
Exam 2
March 5, 2015

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		30	6		9
2		9	7		9
3		9	8		9
4		4	9		12
5		9	Total Score		100

1. (5 points each) Differentiate the following functions. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $r(x) = \pi^2 + e^3$

$$r'(x) = 0$$

B. $k(x) = 5x^2 - 9\sqrt{x} + 3$

$$k'(x) = 10x - \frac{9}{2\sqrt{x}}$$

C. $w(x) = \ln(x) + \arctan(x^3)$

$$w'(x) = \frac{1}{x} + \frac{1}{1+(x^3)^2} \cdot 3x^2$$

D. $v(\theta) = \cos(\tan(\theta))$

$$v'(\theta) = -\sin(\tan(\theta)) \cdot \sec^2(\theta)$$

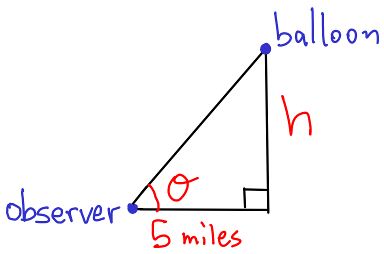
E. $p(t) = \dfrac{\sin(3t)}{2e^t + 7}$

$$p'(t) = \frac{\cos(3t) \cdot 3 \cdot (2e^t + 7) - \sin(3t) \cdot 2e^t}{(2e^t + 7)^2}$$

F. $u(x) = 3^x + \dfrac{6}{x^3}$

$$u'(x) = 3^x \cdot \ln(3) - \frac{18}{x^4}$$

2. (9 points) A hot air balloon rising vertically is tracked by an observer located 5 miles from the lift-off point. At a certain moment, the angle between the observer's line of sight and the horizontal is $\frac{\pi}{4}$, and it is changing at a rate of $\frac{1}{5}$ radians/minute. How fast is the balloon rising at this moment?



Want: $\frac{dh}{dt}$ when $\theta = \frac{\pi}{4}$

Know: $\frac{d\theta}{dt} = \frac{1}{5}\ \frac{\text{rad}}{\text{min}}$ when $\theta = \frac{\pi}{4}$

$$\tan(\theta) = \frac{h}{5}$$

$$h = 5\tan(\theta)$$

$$\frac{dh}{dt} = 5\sec^2(\theta)\frac{d\theta}{dt}.$$

When $\theta = \frac{\pi}{4}$, $\frac{dh}{dt} = 5\sec^2\left(\frac{\pi}{4}\right)\cdot\frac{1}{5} = \sec^2\left(\frac{\pi}{4}\right) = \left(\frac{1}{\cos\left(\frac{\pi}{4}\right)}\right)^2 = \left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2 = 2\ \text{mi}/\text{min}$.

3. (9 points) Let $z(t) = \dfrac{7}{t}$. Using the **limit definition of the derivative**, find $z'(1)$. Make sure to use limit notation correctly.

$$z'(1) = \lim_{h\to 0}\frac{z(1+h) - z(1)}{h} = \lim_{h\to 0}\frac{\frac{7}{1+h} - 7}{h}$$

$$= \lim_{h\to 0}\left[\frac{7}{h(1+h)} - \frac{7}{h}\right] = \lim_{h\to 0}\left[\frac{7}{h(1+h)} - \frac{7(1+h)}{h(1+h)}\right]$$

$$= \lim_{h\to 0}\frac{-7h}{h(1+h)} = \lim_{h\to 0}\frac{-7}{1+h} = \frac{-7}{1+0} = -7$$

4. (4 points) The height of a plane in meters at time t seconds after the plane elevates off of the ground at takeoff is given by $h(t)$. Is $h'(1)$ positive or negative? Briefly explain your answer.

$h'(1)$ is positive. The height of a plane is increasing 1 second after takeoff.

5. (9 points) Find $\dfrac{dy}{dx}$ if $x^2y + 2x^3y = x + y$.

$$\frac{d}{dx}[x^2y + 2x^3y] = \frac{d}{dx}[x+y]$$

$$2xy + x^2\frac{dy}{dx} + 6x^2y + 2x^3\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$x^2\cdot\frac{dy}{dx} + 2x^3\frac{dy}{dx} - \frac{dy}{dx} = 1 - 2xy - 6x^2y$$

$$(x^2+2x^3-1)\frac{dy}{dx} = 1-2xy-6x^2y$$

$$\frac{dy}{dx} = \frac{1-2xy-6x^2y}{x^2+2x^3-1}$$

6. (9 points) Find the derivative of $k(x) = (4x^8+1)^5x^x$.

$$\ln(k(x)) = \ln((4x^8+1)^5x^x) = \ln((4x^8+1)^5) + \ln(x^x)$$

$$= 5\ln(4x^8+1) + x\cdot\ln(x)$$

$$\frac{d}{dx}\ln(k(x)) = \frac{d}{dx}[5\ln(4x^8+1) + x\cdot\ln(x)]$$

$$\frac{k'(x)}{k(x)} = 5\cdot\frac{32x^7}{4x^8+1} + \ln(x) + x\cdot\frac{1}{x}$$

$$k'(x) = k(x)\cdot\left[\frac{160x^7}{4x^8+1} + \ln(x) + 1\right]$$

$$= (4x^8+1)^5x^x\cdot\left[\frac{160x^7}{4x^8+1} + \ln(x) + 1\right]$$

7. (9 points) Find the equation of the tangent line to the curve $y = \dfrac{x}{x+1}$ at $x = 1$.

$$\frac{dy}{dx} = \frac{1\cdot(x+1) - x\cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2}. \qquad \left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{(1+1)^2} = \frac{1}{4}$$

The tangent line at $x=1$ has slope $\frac{1}{4}$ and goes through the point $\left(1, \frac{1}{1+1}\right) = \left(1, \frac{1}{2}\right)$.

Hence, the line is $y - \frac{1}{2} = \frac{1}{4}(x-1)$.

8. (3 points each) The height in feet of a ball t seconds after being thrown vertically is given by $y(t) = -16t^2 + 16t + 5$.

A. Find the velocity of the ball, which is given by $y'(t)$.

$y'(t) = -32t + 16$

B. When is the ball rising?

The ball is rising when $y'(t) > 0$, which happens when $t < \frac{1}{2}$ second.

C. Find the maximum height that the ball reaches.

Since the ball rises until $t = \frac{1}{2}$ second and then starts falling, the maximum height the ball reaches is

$y\left(\frac{1}{2}\right) = -16\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) + 5 = -4 + 8 + 5 = 9$ ft.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	4	1	2
2	5	7	2	6
3	12	9	8	-1
4	21	8	10	-3

9. (4 points each) Using the table above, calculate the following quantities.

A. $a'(1)$ if $a(x) = f(x)g(x)$

$$a'(x) = f'(x)g(x) + f(x)g'(x)$$

$$a'(1) = f'(1)g(1) + f(1)g'(1) = 1\cdot 4 + 3\cdot 2 = 10$$

B. $b'(1)$ if $b(x) = \dfrac{f(x)}{g(x)}$

$$b'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$b'(1) = \frac{f'(1)g(1) - f(1)g'(1)}{[g(1)]^2} = \frac{1\cdot 4 - 3\cdot 2}{4^2} = -\frac{1}{8}$$

C. $c'(1)$ if $c(x) = f(g(x))$

$$c'(x) = f'(g(x))\cdot g'(x)$$

$$c'(1) = f'(g(1))\cdot g'(1) = f'(4)\cdot 2 = 10\cdot 2 = 20$$