Name Solutions Rec. Instr.

Signature Rec. Time

Math 220

Exam 3

April 9, 2015

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		12	7		7
2		8	8		6
3		12	9		10
4		5	10		12
5		6	11		15
6		7	Total Score		100

1. (12 points) Find the absolute maximum and absolute minimum of $k(x) = x^2 - 2x + 5$ on $[-1, 2]$.

$k'(x) = 2x - 2 = 2(x-1)$ is defined everywhere.

$0 = k'(x) = 2(x-1)$ when $x = 1$.

The only critical point in $[-1,2]$ is at $x=1$.

$k(-1) = (-1)^2 - 2(-1) + 5 = 8$

$k(1) = 1^2 - 2(1) + 5 = 4$

$k(2) = 2^2 - 2(2) + 5 = 5$

On $[-1,2]$, $k(x)$ has an absolute max at $(-1, 8)$ and an absolute min at $(1, 4)$.

2. A. (5 points) Find the linearization of $v(x) = \sin(x)$ at $x = 0$.

$v'(x) = \cos(x)$. The linearization of $v(x)$ at $x=0$ is

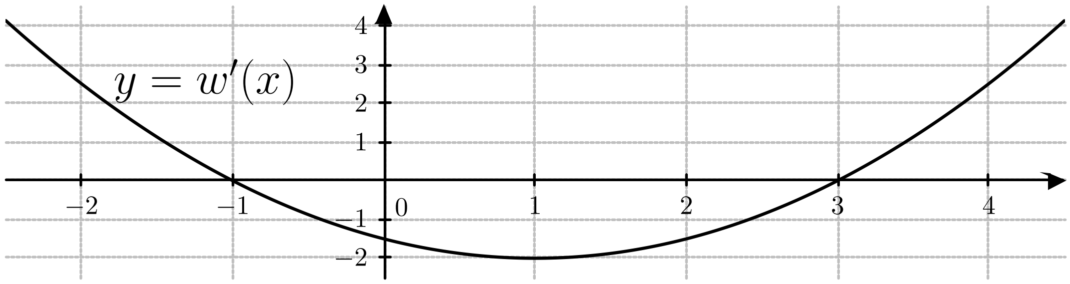
$L(x) = v(0) + v'(0)(x-0) = \sin(0) + \cos(0) \cdot x = x$

B. (3 points) Use your answer from Part **A** to estimate $\sin(-.01)$.

$\sin(-.01) \approx L(-.01) = -.01$

$\uparrow$

$-.01$ is close to 0



3. (3 points each) $y = w'(x)$ is plotted above. Find:

A. Interval(s) where $w(x)$ is increasing: $(-\infty, -1), (3, \infty)$ decreasing: $(-1, 3)$

B. x-coordinate(s) where $w(x)$ has a local max: $x = -1$ local min: $x = 3$

C. Interval(s) where $w(x)$ is concave up: $(1, \infty)$ concave down: $(-\infty, 1)$

D. x-coordinate(s) where $w(x)$ has an inflection point: $x = 1$

4. (5 points) Find the differential dy if $y = e^{3x}$.

$$\frac{dy}{dx} = e^{3x} \cdot 3$$

$$dy = e^{3x} \cdot 3 \cdot dx$$

5. (6 points) Find the most general antiderivative of $\sec^2(x) + 3x^4 + 2$. (I hope that you 'C' what I mean.)

$$\tan(x) + \frac{3}{5}x^5 + 2x + C$$

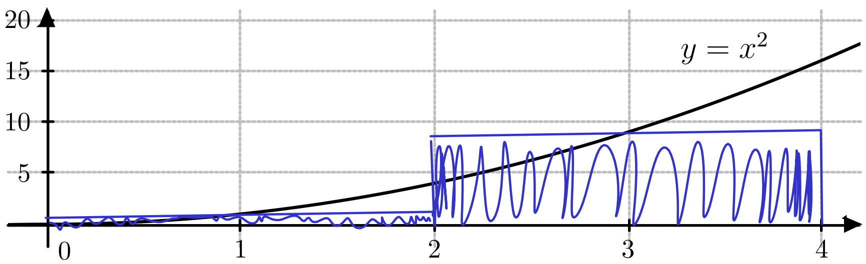
6. (7 points) Find the function $g(x)$ satisfying $g'(x) = \sin(x) + 1$ and $g(0) = 3$.

$$\int (\sin(x) + 1)\,dx = -\cos(x) + x + C$$

$g(x) = -\cos(x) + x + C$ for some constant C.

$3 = g(0) = -\cos(0) + 0 + C = -1 + C$ so $C = 4$

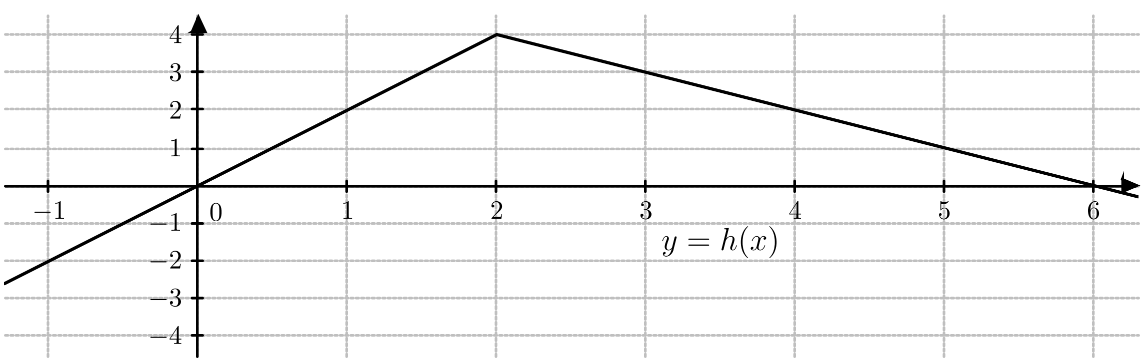
$$g(x) = -\cos(x) + x + 4$$



7. (7 points) Estimate the area between $y = x^2$ and the x-axis over the interval [0,4]. Use $n = 2$ rectangles, taking the sampling points to be midpoints. In the language of our textbook, this is M_2. Also, illustrate the rectangles on the graph above.

$$\Delta x = \frac{4-0}{2} = 2$$

$$M_2 = 2 \cdot 1^2 + 2 \cdot 3^2 = 20$$



8. (3 points each) $y = h(x)$ is plotted above. Evaluate the following definite integrals. (No work needs to be shown.)

A. $\int_{-1}^{0} h(x)\,dx = -\frac{1}{2} \cdot 1 \cdot 2 = -1$

B. $\int_{2}^{6} h(x)\,dx = \frac{1}{2} \cdot 4 \cdot 4 = 8$

9. (10 points) If a bakery charges x dollars per cake, it makes a total profit of $P(x) = -x^2 + 100x - 30$. If the bakery wants to maximize profit, what should it charge per cake? (Justify why your answer is an absolute maximum.)

$P'(x) = -2x + 100$ is defined everywhere.

$0 = P'(x) = -2x + 100$ when $2x = 100$ so $x = 50$.

$x = 50$ is the only critical number.

1st Derivative Justification

Sign of $P'(x)$: $+$ for $x < 50$, $-$ for $x > 50$

$P(x)$ is increasing for $x < 50$ and decreasing for $x > 50$.

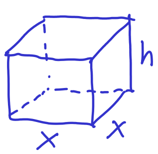
2nd Derivative Justification

$P''(x) = -2 < 0$

$P(x)$ is concave down everywhere.

$P(x)$ is maximized when the bakery charges $x = \$50$ per cake.

10. (12 points) Find the dimensions of the box with square base that has volume 8 and minimal surface area. (Justify why your answer is an absolute minimum.)



Minimize surface area $S = 2x^2 + 4xh$

Volume $x^2h = 8$ so $h = \frac{8}{x^2}$.

Minimize $S(x) = 2x^2 + 4x\left(\frac{8}{x^2}\right) = 2x^2 + \frac{32}{x}$ on $(0, \infty)$.

$S'(x) = 4x - \frac{32}{x^2}$ is defined on $(0, \infty)$.

$S'(x) = 0$ when $4x = \frac{32}{x^2}$ so $x^3 = \frac{32}{4} = 8$ so $x = 2$.

1st Derivative Justification

sign of $S'(x)$: $-$ on $(0, 2)$, $+$ on $(2, \infty)$

$S(x)$ is decreasing on $(0,2)$ and increasing on $(2, \infty)$

2nd Derivative Justification

$S''(x) = 4 + \frac{64}{x^3} > 0$ on $(0, \infty)$

so $S(x)$ is concave up on $(0, \infty)$.

The surface area is minimized when $x = 2$ and $h = \frac{8}{2^2} = 2$.

The dimensions of the optimal box are 2 by 2 by 2.

11. The function $f(x)$ and its first and second derivatives are:

$$f(x) = x^2(x-3) \qquad f'(x) = 3x(x-2) \qquad f''(x) = 6(x-1).$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: $(-\infty, \infty)$

B. (1 point) y-intercept: $(0,0)$

C. (1 point) x-intercept(s): $(0,0)$ and $(3,0)$

D. (1 point) Interval(s) $f(x)$ is increasing: $(-\infty, 0)$ and $(2, \infty)$

E. (1 point) Interval(s) $f(x)$ is decreasing: $(0,2)$

$f'(x) = 0$ at $x = 0, 2$ Sign of $f'(x)$: $+$ | 0 | $-$ | 2 | $+$

F. (1 point) Local maximum(s) (x,y): $(0, 0)$

G. (1 point) Local minimum(s) (x,y): $(2, -4)$

H. (1 point) Interval(s) $f(x)$ is concave up: $(1, \infty)$

I. (1 point) Interval(s) $f(x)$ is concave down: $(-\infty, 1)$

J. (1 point) Inflection point(s) (x,y): $(1, -2)$

K. (5 points) Sketch $y = f(x)$ on the graph below.