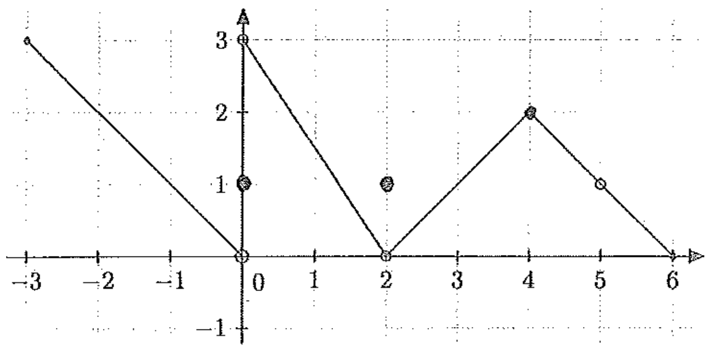
Name *Solutions* Rec. Instr. ______
Signature ______ Rec. Time ______

Math 220
Exam 1
September 22, 2016

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		16	6		27
2		20	7		8
3		5	8		10
4		6			
5		8	Total Score		100



1. (2 points each) Consider the graph of $y = f(x)$ above. State the value of each of the below quantities. If the limit or derivative does not exist, write "does not exist". No work needs to be shown.

A. $\lim_{x \to 0^+} f(x) =$ 3

B. $\lim_{x \to 0} f(x) =$ does not exist

C. $\lim_{x \to 2} f(x) =$ 0

D. $\lim_{x \to 4} f(x) =$ 2

E. $f'(-2) =$ -1

F. $f'(4) =$ does not exist

G. Give all removable discontinuities of $f(x)$: $x =$ 2, 5

H. Give all jump discontinuities of $f(x)$: $x =$ 0

2. (5 points each) Evaluate the following limits. Write ∞ or $-\infty$ for infinite limits, and "does not exist" for limits that do not exist.

A. $\displaystyle\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2}$ $= \displaystyle\lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-1)} = \frac{2+2}{2-1} = 4$

$\dfrac{2^2-4}{2^2-6+2} = \dfrac{0}{0}$ type

B. $\displaystyle\lim_{t\to 3^-} \frac{1-2t}{2t-6}$ $= \displaystyle\lim_{t\to 3^-} \frac{1-2t}{2} \cdot \frac{1}{(t-3)}$

A/0 type

$= -\dfrac{5}{2} \displaystyle\lim_{t\to 3^-} \frac{1}{t-3} = -\frac{5}{2}(-\infty) = +\infty$

$t = 2.9999$ $\quad \dfrac{1}{-.0001}$

C. $\displaystyle\lim_{x\to 4} \frac{x^2-9}{x-2}$ $= \dfrac{4^2-9}{4-2} = \dfrac{7}{2}$

Can be done by direct substitution

D. $\displaystyle\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$ $= \displaystyle\lim_{h\to 0} \frac{(\sqrt{9+h}-3)}{h} \frac{(\sqrt{9+h}+3)}{\sqrt{9+h}+3}$

$\dfrac{\sqrt{9}-3}{0} = \dfrac{0}{0}$ type $\quad = \displaystyle\lim_{h\to 0} \frac{\cancel{(9+h)-9}}{h(\sqrt{9+h}+3)} = \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3}$

$= \dfrac{1}{\sqrt{9}+3} = 1/6$

3. (5 points) Let $f(x) = x^3 + x + 1$. Use the Intermediate Value Theorem to show that there is a point c between -1 and 0 such that $f(c) = 0$. Give a careful explanation to receive full credit.

$f(-1) = (-1)^3 - 1 + 1 = -1$, $f(0) = 0 + 0 + 1 = 1$

Since $f(-1) < 0$, $f(0) > 0$ and $f(x)$ is continuous on $[-1, 0]$

IVT implies that $f(c) = 0$ for some c in $(-1, 0)$.

4. (6 points) Let $f(x) = \begin{cases} x^2 - x & \text{if } x < 2 \\ x^2 + x + c & \text{if } x \geq 2 \end{cases}$

A. Find $\lim_{x \to 2^-} f(x)$

$$= \lim_{x \to 2^-} x^2 - x = 2^2 - 2 = 2$$

B. Find the value of c so that $f(x)$ is continuous on $\mathbb{R}$.

We need $\lim_{x \to 2^-} x^2 - x = f(2)$

$$2^2 - 2 = 2^2 + 2 + c$$

$$2 = 6 + c$$

$$c = -4$$

5. (8 points) Use the limit definition of derivative to find $f'(x)$ where $f(x) = 3x^2 - x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - (x+h) - [3x^2 - x]}{h}$$

$$= \lim_{h \to 0} \frac{3(\cancel{x^2} + 2hx + h^2) - \cancel{x} - h - \cancel{3x^2} + \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{6hx + 3h^2 - h}{h} = \lim_{h \to 0} 6x + 3h - 1$$

$$= 6x - 1$$

6. Calculate the derivative of the following functions. Use the rules for derivatives, not the limit definition of the derivative. You do not need to simplify your answer.

$x^{1/2}$

A. (6 points) $w(x) = x^7 - 2\sqrt{x} - 5$

$w'(x) = 7x^6 - 2 \cdot \frac{1}{2} x^{-1/2}$

$= 7x^6 - x^{-1/2}$

B. (7 points) $v(x) = \sin^2(x^3 + 4x)$ $= [\sin(x^3+4x)]^2$

$v'(x) = 2\sin(x^3+4x) \cdot \cos(x^3+4x) \cdot (3x^2+4)$

C. (7 points) $h(x) = x^3 \tan(2x)$

$h'(x) = x^3 \sec^2(2x) \cdot 2 + \tan(2x) \cdot 3x^2$

D. (7 points) $g(x) = \frac{x-1}{x^2+x}$

$$g'(x) = \frac{(x^2+x) \cdot 1 - (x-1)(2x+1)}{(x^2+x)^2}$$

7. (8 points) Find the equation of the tangent line to the curve $y = x^2 + 3x$ at the point where $x = 1$.

$y' = 2x + 3$, $y'|_{x=1} = 2\cdot 1 + 3 = 5$

Point: $x = 1$, $y = 1^2 + 3 = 4$

pt-slope: $y - y_0 = m(x - x_0)$

$y - 4 = 5(x - 1)$

or $y = 5x - 1$

Both are good

8. An object starting at $t = 0$ is traveling along a horizontal line with position function $s(t) = 8t^2 - \frac{2}{3}t^3$, with s measured in feet and t in seconds.

A. (3 points) Find the velocity of the object at time $t = 2$ seconds. Include the correct units.

$v(t) = s'(t) = 16t - \frac{2}{3}\cdot 3t^2 = 16t - 2t^2$

$v(2) = 16\cdot 2 - 2\cdot 4 = 24$ ft/sec

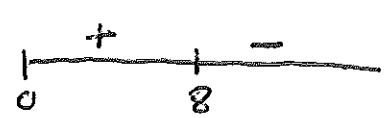
B. (3 points) Find the acceleration of the object at time $t = 2$ seconds. Include the correct units.

$a(t) = v'(t) = 16 - 4t$

$a(2) = 16 - 8 = 8$ ft/sec^2

C. (2 points) Determine the time interval when the object is moving to the right, that is, in the direction of increasing s.

$v(t) > 0$, $16t - 2t^2 > 0$, $2t(8 - t) > 0$



$0 < t < 8$ sec

D. (2 points) At time $t = 6$ seconds, is the object speeding up or slowing down? Explain.

$v(6) = 16\cdot 6 - 2\cdot 6^2 = 96 - 72 = 24 > 0$ moving right

$a(6) = 16 - 4\cdot 6 = 16 - 24 = -8 < 0$

v and a have opposite signs, so it is slowing down