Name Solutions Rec. Instr.

Signature Rec. Time

Math 220

Exam 2

October 20, 2016

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work.**

Problem	Points	Points Possible	Problem	Points	Points Possible
1		13	6		8
2		8	7		10
3		9	8		18
4		8	9		5
5		6	10		15

Total =

1. Differentiate the following functions. You do not need to simplify your answers.

A. (7 points) $w(x) = x^3 e^{1/x}$ $\leftarrow x^{-1}$

$$w'(x) = x^3 e^{1/x} (-1) x^{-2} + e^{1/x} \cdot 3x^2 \leftarrow \text{Good enough}$$

$$= -x e^{1/x} + 3x^2 e^{1/x}$$

B. (6 points) $h(x) = \tan^{-1}(x^2)$.

$$h'(x) = \frac{1}{1+(x^2)^2} \cdot 2x = \frac{2x}{1+x^4}$$

Good enough

2. (8 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = x^{2x}(1-x)^7$.

$$\ln y = \ln x^{2x} + \ln (1-x)^7$$

$$\ln y = 2x \ln x + 7 \ln (1-x)$$

$$\Rightarrow \frac{1}{y} \cdot y' = 2x \cdot \frac{1}{x} + 2 \ln x + \frac{7}{1-x}(-1)$$

$$= 2 + 2\ln x - \frac{7}{1-x}$$

$$\Rightarrow y' = x^{2x}(1-x)^7 \left[2 + 2\ln x - \frac{7}{1-x}\right]$$

3. (9 points) Use implicit differentiation to find an equation for the tangent line to the curve $xy + 7 = x^3 + y^3$ at the point $(2, 1)$.

$xy' + y = 3x^2 + 3y^2 y'$

$xy' - 3y^2 y' = 3x^2 - y$

$y'(x - 3y^2) = 3x^2 - y$

$y' = \dfrac{3x^2 - y}{x - 3y^2}$

At $(2,1)$

$y' = \dfrac{3 \cdot 2^2 - 1}{2 - 3} = \dfrac{11}{-1} = -11$

Pt-Slope Form:

$y - 1 = -11(x - 2)$

or $y = -11x + 23$

4. A. (5 points) Find the linear approximation of $f(x) = \sqrt[3]{x}$ near $x = 8$.

$f'(x) = \frac{d}{dx} x^{1/3} = \frac{1}{3} x^{\frac{1}{3} - 1} = \frac{1}{3} x^{-2/3} = \dfrac{1}{3x^{2/3}}$

$f'(8) = \dfrac{1}{3 \cdot 8^{2/3}} = \dfrac{1}{3 \cdot 2^2} = \dfrac{1}{12}$, $f(8) = \sqrt[3]{8} = 2$

$L(x) = f(a) + f'(a)(x - a)$

$L(x) = 2 + \frac{1}{12}(x - 8)$

B. (3 points) Use your answer from Part A to estimate $\sqrt[3]{8.1}$. (No need to simplify your answer.)

$L(8.1) = 2 + \frac{1}{12}(8.1 - 8)$

$= 2 + \frac{1}{12} \cdot \frac{1}{10}$

$= 2 + \dfrac{1}{120}$

5. (6 points) The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. Estimate the change in volume by calculating dV, given that $r = 3$ inches and $dr = \frac{1}{12\pi}$ inches.

$$dV = \frac{dV}{dr} \cdot dr = 4\pi r^2 \, dr = 4\pi \, 3^2 \cdot \frac{1}{12\pi} = 3 \text{ in}^3$$

6. (8 points) Determine the absolute minimum and absolute maximum value of the function $f(x) = x^3 - 3x$ over the interval $[0, 2]$.

$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$ Critical pts: $x = \pm 1$, only $x = 1$ is in the interval.

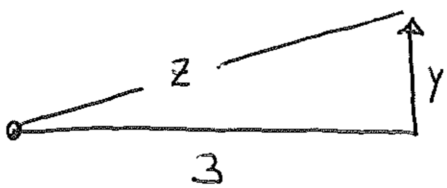
C.P. and end pts

x	$x^3 - 3x$
0	0
1	$1 - 3 = -2$
2	$8 - 6 = 2$

Absolute min value is -2, at $x = 1$

Absolute max value is 2, at $x = 2$

7. (10 points) A rocket is launched vertically upward from a point 3 miles east of an observer on the ground. Use related rates to determine the rate that the distance between the rocket and the observer is increasing when the rocket is 4 miles above the ground and traveling 500 miles per hour (vertically upward).



Given $dy/dt = 500$ mi/hr when $y = 4$,
Find dz/dt when $y = 4$ mi.

$$z^2 = 3^2 + y^2$$
$$\Rightarrow \frac{d}{dt} z^2 = \frac{d}{dt}(9 + y^2)$$
$$\Rightarrow 2z \, dz/dt = 2y \, dy/dt$$
$$\Rightarrow dz/dt = \frac{y}{z} \cdot dy/dt$$

When $y = 4$, $z^2 = 3^2 + 4^2 = 25$
$z = 5$

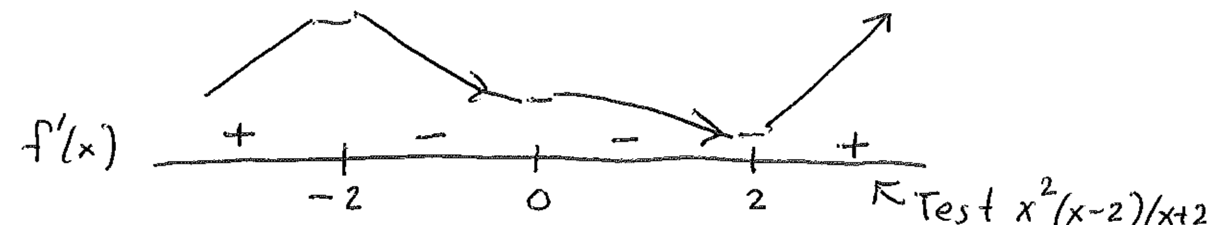
Thus $\frac{dz}{dt} = \frac{4}{5} \cdot 500$ mi/hr
$= 400$ mi/hr.

8. Let $g(x) = x^5 - \frac{20}{3}x^3$.

A. (4 points) Find the critical points of $g(x)$ and draw a number line showing where $g'(x)$ is positive and negative.

$g'(x) = 5x^4 - 20x^2 = 5x^2(x^2-4) = 5x^2(x-2)(x+2)$

Critical pts: $x = 0, 2, -2$



B. (2 points) Give the open interval(s) where $g(x)$ is increasing:

$(-\infty, -2), (2, \infty)$

C. (3 points) Classify each critical point as a local minimum, local maximum or neither.

$x = -2$ is a local max

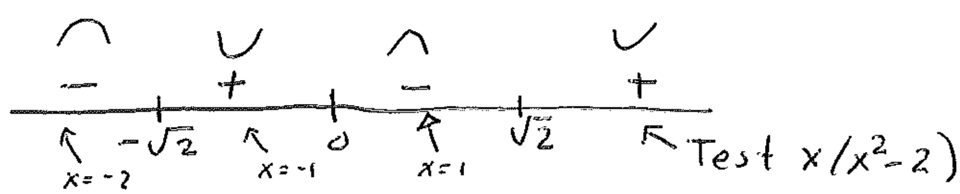
$x = 0$ is neither

$x = 2$ is a local min

D. (4 points) Draw a number line showing where $g''(x)$ is positive and negative.

$g''(x) = 20x^3 - 40x = 20x(x^2-2)$

Critical pts: $x = 0, \sqrt{2}, -\sqrt{2}$



E. (2 points) Give the open interval(s) where $g(x)$ is concave up:

$(-\sqrt{2}, 0), (\sqrt{2}, \infty)$

F. (3 points) Give the x-coordinates of all inflection points of $g(x)$:

$-\sqrt{2}, 0, \sqrt{2}$

9. (5 points) Evaluate $\lim_{x\to-\infty}\frac{x+3}{\sqrt{x^2-1}}$ $= \lim_{x\to-\infty} \dfrac{x+3}{\sqrt{x^2(1-1/x^2)}}$

$$= \lim_{x\to-\infty} \frac{x+3}{-x\sqrt{1-\frac{1}{x^2}}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

Note: $\sqrt{x^2} = -x$ since $x < 0$

$$= \lim_{x\to-\infty} \frac{1+3/x}{-\sqrt{1-1/x^2}} = \frac{1+0}{-\sqrt{1-0}} = \frac{1}{-1} = -1$$

10. Let $f(x) = \frac{2x^2-6x}{4-x^2}$. Given: $f'(x) = \frac{-2(3x^2-8x+12)}{(4-x^2)^2}$.

A. (2 points) Give the equation of the horizontal asymptote for $f(x)$ (if any).

$y = 2/_{-1} = -2$ (ratio of leading coeff.)

B. (2 points) Give the equations of all vertical asymptotes for $f(x)$ (if any).

$x = 2,\ x = -2$

C. (2 points) Give the x-intercepts (if any): $2x(x-3) = 0,\ x = 0, 3$

D. (1 point) Give the y-intercept (if any): $x = 0,\ y = \frac{0}{4} = 0$

E. (2 points) Find all critical points of $f(x)$ (if any).

$3x^2 - 8x + 12 = 0 \Rightarrow x = (8 \pm \sqrt{64 - 4\cdot3\cdot12})/6 \rightarrow$ Not real $\sqrt{(-)}$

denom $= 0$ at asymptotes. Thus no critical pts.

F. (2 points) Show that $f(x)$ is always decreasing on its domain.

$f'(x)$ — -2 — 2 —

Test $\dfrac{-2(3x^2-8x+12)}{(4-x^2)^2}$ quadratic is always positive; denom is positive.

G. (4 points) Make a rough sketch of the graph of $f(x)$, including the asymptotes and intercepts. Do not worry about plotting inflection points.