Name Solutions Rec. Instr.
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Math 220
Exam 3
November 17, 2016

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. **Show your work.**

Problem	Points	Points Possible	Problem	Points	Points Possible
1		18	6		6
2		18	7		6
3		10	8		8
4		8	9		6
5		10	10		10

Total Score:

1. Evaluate the following integrals.

A. (6 points) $\int \frac{x^2 - 7x}{x^3}\, dx = \int \frac{1}{x} - 7x^{-2}\, dx$

$= \ln|x| - 7\frac{x^{-1}}{-1} + C$

$= \ln|x| + 7x^{-1} + C$

B. (6 points) $\int \sqrt{\tan x}\, \sec^2 x\, dx$

$u = \tan x$

$du = \sec^2 x\, dx$

$= \int u^{1/2}\, du$

$= \frac{2}{3} u^{3/2} + C = \frac{2}{3} (\tan x)^{3/2} + C$

C. (6 points) $\int \frac{(\ln x)^3}{x}\, dx$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$= \int u^3\, du = \frac{u^4}{4} + C = \frac{1}{4}(\ln x)^4 + C$

2. Evaluate the following limits.

A. (6 points) $\lim_{t\to 0} \frac{4t - \sin(2t)}{5t^2 + 3t}$ $=$ $\frac{0}{0}$ -type limit, so we use L'Hopital

$$= \lim_{t\to 0} \frac{4 - 2\cos(2t)}{10t + 3}$$

$$= \frac{4 - 2\cos(0)}{10 \cdot 0 + 3} = \frac{2}{3}$$

B. (6 points) $\lim_{x\to\infty} x \sin\left(\frac{2}{x}\right)$ $= \infty \cdot 0$

$$= \lim_{x\to\infty} \frac{\sin\left(\frac{2}{x}\right)}{1/x} = \frac{0}{0}\text{-type, so we can use L'Hopital}$$

$$= \lim_{x\to\infty} \frac{\cos\left(\frac{2}{x}\right)(-2)\cancel{x^{-2}}}{-1\,\cancel{x^{-2}}} = \lim_{x\to\infty} \cos\left(\frac{2}{x}\right)\cdot 2$$

$$= \cos(0)\cdot 2 = 2$$

C. (6 points) $L = \lim_{x\to\infty} (5x)^{1/x}$ $= \infty^0$

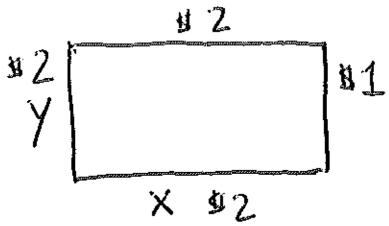
$$\ln L = \lim_{x\to\infty} \ln(5x)^{1/x} = \lim_{x\to\infty} \frac{1}{x}\ln(5x)$$

$$= \lim_{x\to\infty} \frac{\ln(5x)}{x} = \frac{\infty}{\infty}\text{-type, so use L'Hopital}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{\cancel{5}x}\cdot\cancel{5}}{1} = 0$$

Thus $L = e^0 = 1$

3. (10 points) A rectangular fence consists of three sides costing \$2 per meter, and one side costing \$1 per meter If the area of the rectangle is 12 square meters, find the dimensions that minimize the cost of the fence.



Given $A = \text{area} = 12\ m^2$,

Minimize cost $C = 2x + 2x + 2y + y$

$C = 4x + 3y$

Given $xy = 12 \Rightarrow y = \frac{12}{x}$, $\quad C = 4x + 3 \cdot \frac{12}{x} = 4x + 36x^{-1}$

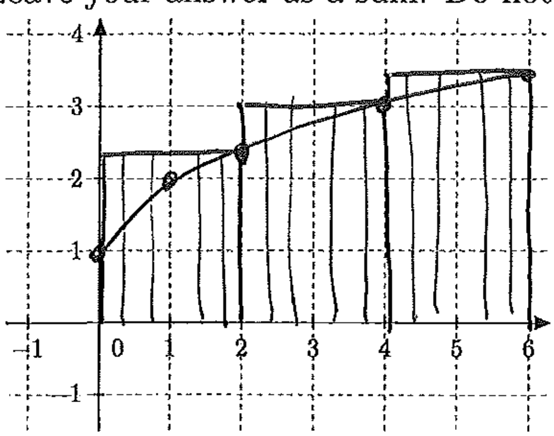
Critical pts: $\frac{dC}{dx} = 4 - 36x^{-2} = 0 \Rightarrow \frac{36}{x^2} = 4 \Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$ only $x = 3$ makes sense.

Thus $x = 3\,m$, $y = \frac{12}{3} = 4\,m$.

Note: This is a minimum since $C \to \infty$ as $x \to 0$ and $C \to \infty$ as $x \to \infty$.

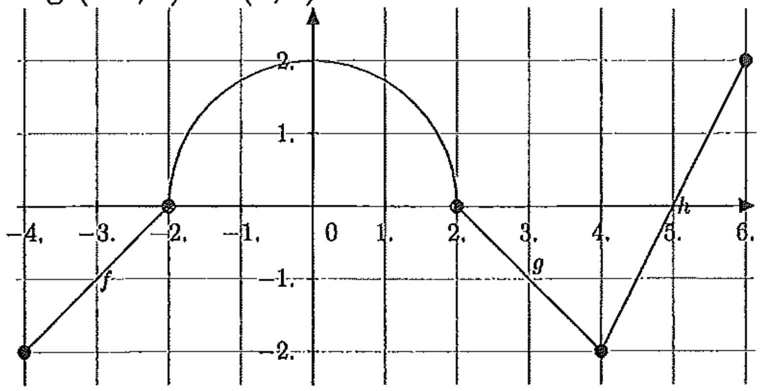
4. (8 points) Estimate the area below the curve $y = \sqrt{x} + 1$ over the interval [0,6] using R_3, the right end point approximation with three intervals. Also, make a sketch of the graph of $y = \sqrt{x} + 1$ and illustrate the rectangles on your graph. (Leave your answer as a sum. Do not simplify. $\sqrt{2} \approx 1.41$, $\sqrt{6} \approx 2.45$.)



$R_3 = 2 \cdot (\sqrt{2} + 1) + 2 \cdot (\sqrt{4} + 1) + 2(\sqrt{6} + 1)$

$= 2(\sqrt{2} + 1) + 6 + 2(\sqrt{6} + 1)$

5. For the function $f(x)$ graphed below evaluate the given integrals. The arc joining $(-2,0)$ to $(2,0)$ is a semicircle.



A. (5 points) $\int_{-2}^{2} f(x)\,dx$ = Area semi-circle $= \frac{1}{2}\pi\cdot 2^2 = 2\pi$

B. (5 points) $\int_{-2}^{6} f(x)\,dx = \frac{1}{2}\pi\cdot 2^2 - \frac{1}{2}3\cdot 2 + \frac{1}{2}1\cdot 2$

$= 2\pi - 3 + 1 = 2\pi - 2$

6. (6 points) Evaluate the following integral. Use principles of symmetry.

$$\int_{-2}^{2} \frac{\sin x}{1+x^2} + \cos\left(\tfrac{\pi}{4}x\right)\,dx = 2\int_0^2 \cos\left(\tfrac{\pi}{4}x\right)\,dx$$

odd function integral = 0

even

$$= 2\sin\left(\tfrac{\pi}{4}x\right)\frac{4}{\pi}\Big|_0^2$$

$$= 2\sin\left(\tfrac{\pi}{2}\right)\cdot\frac{4}{\pi} - 0 = \frac{8}{\pi}$$

7. (6 points) Find the average value of $f(x) = \sqrt{x}$ over the interval $[0,4]$.

$$f_{ave} = \frac{1}{4}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\frac{2}{3}x^{3/2}\Big|_0^4 = \frac{1}{6}4^{3/2} - 0$$

$$= \frac{8}{6} = \frac{4}{3}$$

8. (8 points) Solve the initial value problem for $f(t)$: $f'(t) = 4e^{3t}$, $f(0) = 5$.

$$f(t) = \int 4e^{3t}\,dt = 4e^{3t} \cdot \frac{1}{3} + C = \frac{4}{3}e^{3t} + C$$

$$f(0) = 5 \Rightarrow \frac{4}{3}e^{0} + C = 5 \Rightarrow \frac{4}{3} + C = 5 \Rightarrow C = 5 - \frac{4}{3} = \frac{15}{3} - \frac{4}{3} = 11/3$$

$$f(t) = \frac{4}{3}e^{3t} + \frac{11}{3}$$

9. (3 points) **A.** Find $\dfrac{d}{dx}\displaystyle\int_2^x \frac{\sin t}{1+t}\,dt = \dfrac{\sin x}{1+x}$

(3 points) **B.** Find $\dfrac{d}{dx}\displaystyle\int_2^{x^3} \frac{\sin t}{1+t}\,dt = \dfrac{\sin(x^3)}{1+x^3} \cdot 3x^2$

10. An object moves along a straight line with velocity $v(t) = 4 - t^2$, m/sec.

A. (5 points) Find the displacement of the object over the time interval $[0, 3]$ seconds.

$$s(3) - s(0) = \int_0^3 4 - t^2\,dt = 4t - \frac{t^3}{3}\Big|_0^3 = 4 \cdot 3 - \frac{3^3}{3} - 0 = 12 - 9 = 3\ m$$

B. (5 points) Find the total distance the object travels over the time interval $[0, 3]$ seconds.

$v(t) = (2-t)(2+t)$

right $+$ on $[0, 2]$; left $-$ on $[2, 3]$

$$\text{Right} = \int_0^2 4 - t^2\,dt = 4t - \frac{t^3}{3}\Big|_0^2 = 8 - \frac{8}{3} - 0 = \frac{16}{3}\ m$$

$$\text{Left} = \int_2^3 4 - t^2\,dt = 4t - \frac{t^3}{3}\Big|_2^3 = (12 - 9) - \left(8 - \frac{8}{3}\right) = 3 - \frac{16}{3} = \frac{-7}{3}$$

$$\text{Total distance} = \frac{16}{3} + \frac{7}{3} = \frac{23}{3}\ m$$