Name ______________________ Rec. Instr. ______________________
Signature ______________________ Rec. Time ______________________

Math 220
Final Exam
December 14, 2016

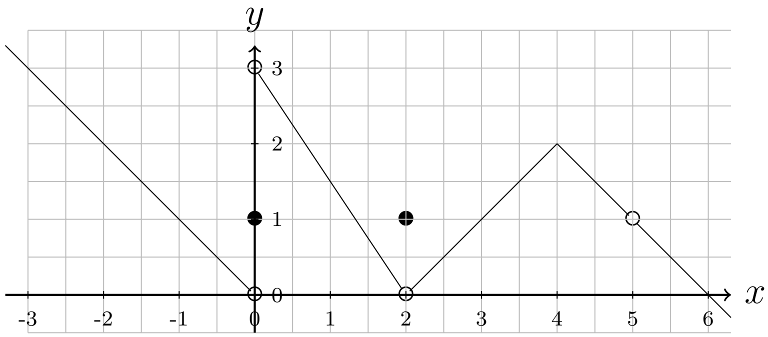
No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 1 hour and 50 minutes to complete the exam.

Total = 200 points. **Show your work unless stated otherwise.**

Problem	Points	Points Possible	Problem	Points	Points Possible
1		10	9		12
2		18	10		10
3		8	11		24
4		24	12		8
5		12	13		12
6		10	14		8
7		10	15		8
8		10	16		16

Total Score:

1. (2 points each) Evaluate the following for the graph below or state they do not exist. No work needs to be shown.



a. Find $\lim_{x \to 0^+} f(x) =$

b. Find $\lim_{x \to 2} f(x) =$

c. Indicate all values of x at which $f'(x)$ is not defined.

d. Indicate all values of x at which $f(x)$ is not continuous.

e. Find $f'(1) =$

2. (6 points each) Evaluate the following limits.

a. $\lim_{x \to 3} \dfrac{x-3}{9x - x^3} =$

b. $\lim_{h \to 0} \dfrac{\tan(2h)}{\sin(5h)} =$

c. $\lim_{x \to \infty} (5+x)^{1/x}$

3. (8 points) Use the definition of derivative as a limit to find $f'(x)$ for $f(x) = 3x^2 - x$.

4. (8 points each) Compute the following derivatives. **DO NOT SIMPLIFY**

a. $f'(t)$ where $f(t) = \cos^2(2t + 1)$.

b. $\dfrac{d}{dx}\, x \ln(x^2 + 2)$

c. $\dfrac{d}{dx}\, \dfrac{e^{5x}}{x^2 + 1} =$

5. (4 points each) Let $f(x) = x^2(x - 4)^3$. Given: $f'(x) = x(x - 4)^2(5x - 8)$.

a. Find the critical points of $f(x)$.

b. Find the open intervals where $f(x)$ is increasing and decreasing.

c. Classify each critical point as a local minimum, local maximum or neither.

6. Let $g(x) = 3x^5 + 20x^3$.

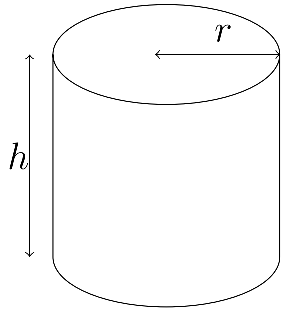
a. (6 points) Determine the open intervals where $g(x)$ is concave up and concave down.

b. (4 points) Determine all inflection points of $g(x)$. Just give the x-coordinates.

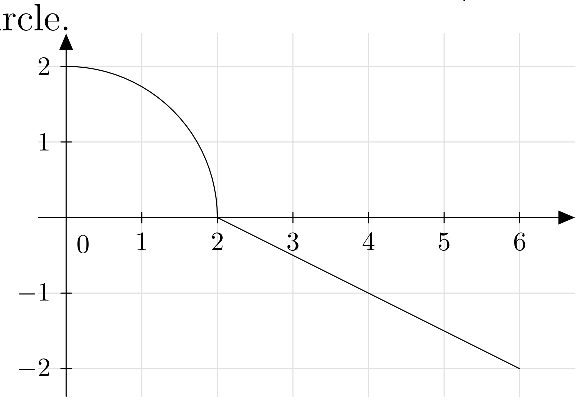
7. (10 points) Use implicit differentiation to find the equation of the tangent line to the curve $x^3 + y^2 = 5y + 4$ at $(2, 1)$.

8. (10 points) Consider a right triangle with edges of length x, y, z, with z the hypotenuse. If x is increasing at a rate of 5 m/sec and z is increasing at a rate of 7 m/sec, at what rate is y increasing when $x = 3$ m and $z = 5$ m?

9. (12 points) Find the dimensions of a cylinder with total surface area 6π square meters, including top and bottom, that maximizes its volume. (Recall, $V = \pi r^2 h$ and the side wall of the cylinder has area $2\pi rh$.)



10. The velocity function $v = v(t)$ for an object moving along a straight line is graphed below. The horizontal axis is time measured in seconds, and the vertical axis is velocity in m/sec. The arc from (0,2) to (2,0) is a quarter circle.



a. (5 points) Let $s = s(t)$ denote the position of the object. If the object is at position $s = 3$ when $t = 0$, where is it after 6 seconds?

b. (5 points) Find the total distance the object travels during the time interval $[0, 6]$ seconds.

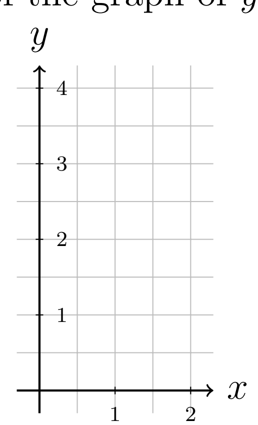
11. (8 points each) Evaluate the following integrals.

a. $\displaystyle\int \sin(\pi x/2) + 2^x - \frac{1}{\sqrt{1-x^2}}\,dx$

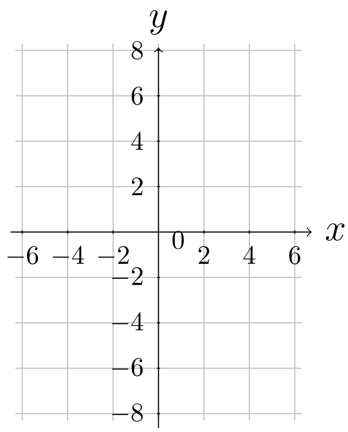
b. $\displaystyle\int \tan^3(2x)\ \sec^2(2x)\ dx$

c. $\displaystyle\int_0^1 \frac{x+2}{x^2+4x+1}\,dx$

12. (8 points) Estimate the area below the curve $y = x^2$ over the interval [0,2] using L_4, the left end point approximation with four rectangles. Also, make a sketch of the graph of $y = x^2$ and illustrate the rectangles on your graph.



13. (12 points) Make a sketch of the region bounded between the parabola $y = 8 - x^2$ and the line $y = x + 2$, and then calculate its area.

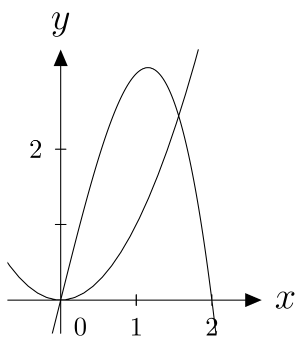


14. (8 points) Solve the initial value problem: $f'(t) = \sqrt{t}$, $f(1) = 2$.

15. (8 points) a) Find the linear approximation of $f(x) = \sqrt{x}$ near $x = 4$.

b) Use your estimate in part a) to estimate $\sqrt{4.1}$.

16. Below is a sketch of the region bounded between the curves $y = 4x - x^3$ and $y = x^2$ for $x \geq 0$. Set up integrals for the following volumes but **do not evaluate the integrals.**



a. (4 points) Start by finding the point of intersection of the two curves with $x > 0$. Just give the x-coordinate.

b. (6 points) The volume of the solid obtained by rotating the region around the x-axis.

c. (6 points) The volume of the solid obtained by rotating the region around the y-axis.