Name Solutions ______________ Rec. Instr. ______________
Signature ______________ Rec. Time ______________

Math 220
Exam 1
February 4, 2016

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

SHOW YOUR WORK!

Problem	Points	Points Possible	Problem	Points	Points Possible
1		12	7		4
2		12	8		6
3		16	9		8
4		8	10		8
5		8	11		10
6		8	Total Score		100

1. (4 points each) Evaluate the following limits.

A. $\lim_{\theta \to 0} \frac{\cos(\theta)}{\theta^2 + 3}$ $= \frac{\cos(0)}{0^2+3} = \frac{1}{3}$

B. $\lim_{\theta \to 0} \frac{7\sin(\theta)}{3\theta}$ $= \frac{7}{3} \lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = \frac{7}{3} \cdot 1 = \frac{7}{3}$

C. $\lim_{x \to \infty} \frac{4 - 3x^2 + 5x^4}{-2x^4 + 3x + 9}$ $= \frac{5}{-2} = -\frac{5}{2}$

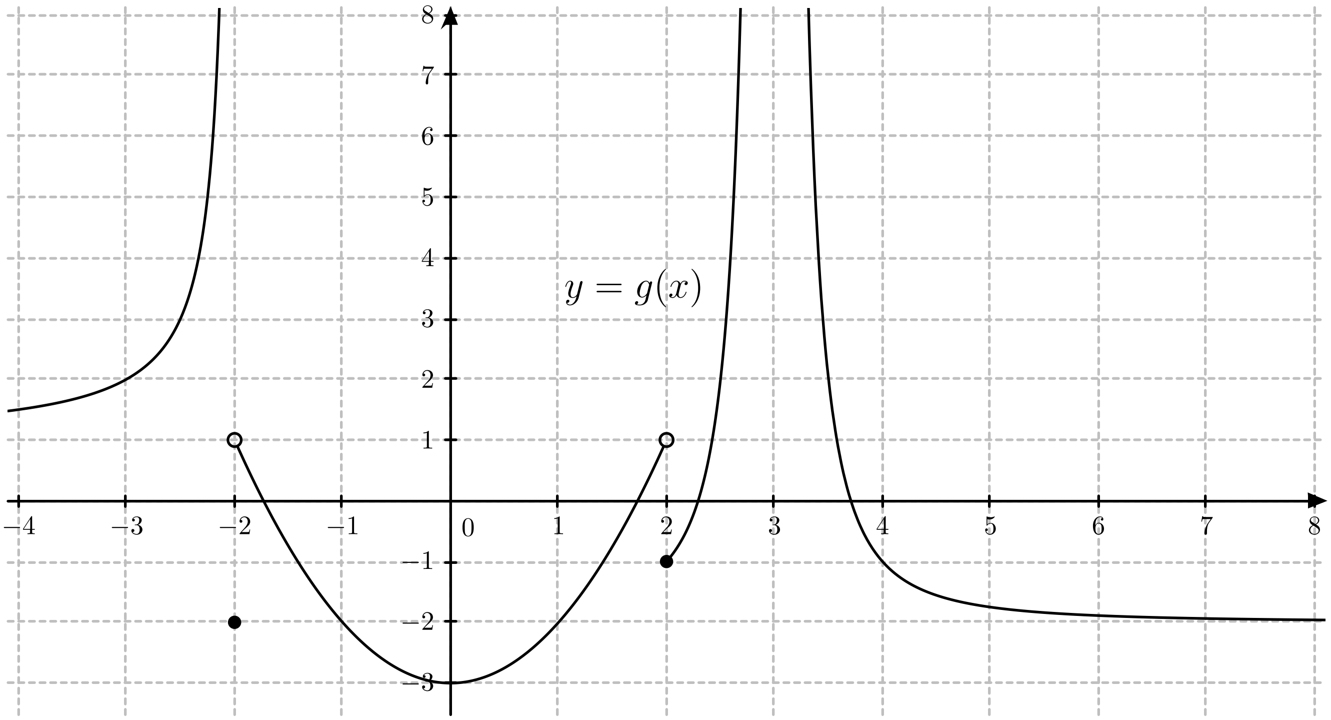
2. (6 points each) Evaluate the following limits.

A. $\lim_{w \to 3} \frac{w^2 - 2w - 3}{w - 3}$ $= \lim_{w \to 3} \frac{(w-3)(w+1)}{w-3} = \lim_{w \to 3} (w+1) = 4$

B. $\lim_{t \to 1} \left(\frac{1}{t^2 - t} - \frac{1}{t - 1} \right)$ $= \lim_{t \to 1} \left(\frac{1}{t(t-1)} - \frac{1}{(t-1)} \right) = \lim_{t \to 1} \left(\frac{1}{t(t-1)} - \frac{t}{t(t-1)} \right)$

$= \lim_{t \to 1} \frac{1-t}{t(t-1)} = \lim_{t \to 1} \frac{-(t-1)}{t(t-1)} = \lim_{t \to 1} \frac{-1}{t}$

$= -\frac{1}{1} = -1$



3. (2 points each) Consider the graph of $y = g(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x\to 2^-} g(x) = 1$

B. $\lim_{x\to 2^+} g(x) = -1$

C. $\lim_{x\to 2} g(x)$ does not exist

D. $g(2) = -1$

E. $\lim_{x\to -3} g(x) = 2$

F. $\lim_{x\to 3} g(x) = +\infty$
(or does not exist)

G. $\lim_{x\to \infty} g(x) = -2$

H. $\lim_{x\to -2^+} g(x) = 1$

4. (8 points each) Provided that $-2x^4+5 \le h(x) \le 11-8x$ for all x, find $\lim_{x\to 1} h(x)$. (**Justify your reasoning, and state the name of any theorem used.**)

$\lim_{x\to 1} (-2x^4+5) = -2\cdot(1)^4+5 = 3$

$\lim_{x\to 1} (11-8x) = 11-8\cdot 1 = 3.$

Since $-2x^4+5 \le h(x) \le 11-8x$ for all x, by the Squeeze Theorem, $\lim_{x\to 1} h(x) = 3.$

5. Suppose that an object is at position $s(t) = \sqrt{t}$ feet at time t seconds.

A. (3 points) Find the average velocity of the object over a time interval from time 4 seconds to time $4+h$ seconds.

average velocity from 4 sec to 4+h sec $= \dfrac{s(4+h)-s(4)}{(4+h)-4} = \dfrac{\sqrt{4+h}-\sqrt{4}}{h} = \dfrac{\sqrt{4+h}-2}{h}\ \dfrac{\text{ft}}{\text{sec}}$

B. (5 points) Find the instantaneous velocity of the object at time 4 seconds by taking the limit of the average velocity in Part A as $h \to 0$.

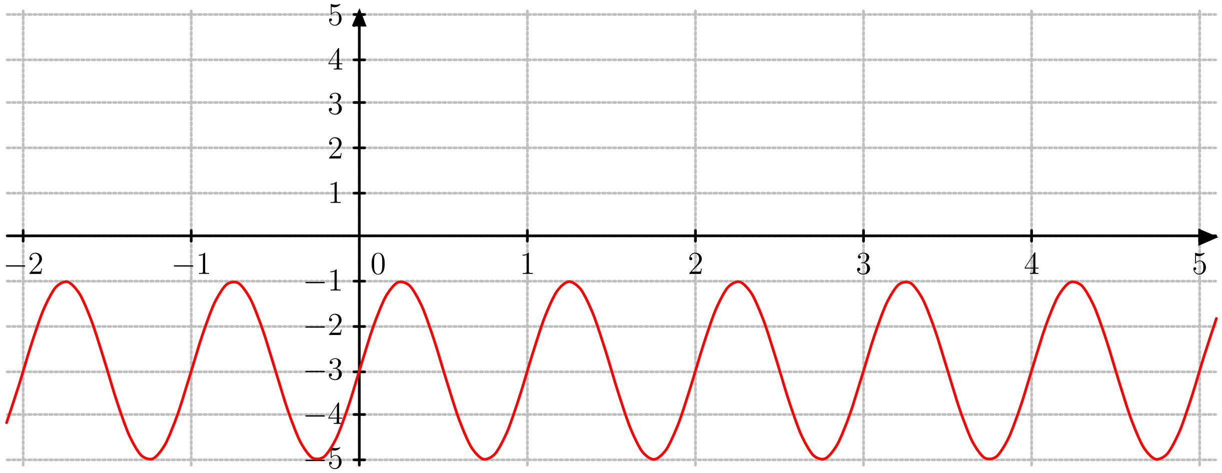
instantaneous velocity at 4 sec $= \lim_{h\to 0} \dfrac{\sqrt{4+h}-2}{h} = \lim_{h\to 0}\left(\dfrac{\sqrt{4+h}-2}{h}\cdot\dfrac{-\sqrt{4+h}-2}{-\sqrt{4+h}-2}\right)$

$= \lim_{h\to 0} \dfrac{-(4+h)+4}{h(-\sqrt{4+h}-2)} = \lim_{h\to 0} \dfrac{-h}{h(-\sqrt{4+h}-2)}$

$= \lim_{h\to 0} \dfrac{-1}{-\sqrt{4+h}-2} = \dfrac{-1}{-\sqrt{4+0}-2} = \dfrac{-1}{-4}$

$= \dfrac{1}{4}\ \dfrac{\text{ft}}{\text{sec}}$

6. (8 points) Sketch the graph of $y = 2\sin(2\pi x) - 3$.



7. (4 points) Find functions $b(x)$ and $c(x)$ such that $b(c(x)) = \sqrt{\cos(x) + 1}$.

$b(x) = \sqrt{x}$ $\qquad\qquad$ $c(x) = \cos(x) + 1$

8. (6 points) Write an equation for the line with slope -5 that goes through the point (1,7).

$y - 7 = -5(x - 1)$

[Alternatively, $y = -5x + 12$]

9. (4 points each) Given that $\lim_{x\to 3} h(x) = 4$ and $\lim_{x\to 3} k(x) = 5$, find the following limits.

A. $\lim_{x\to 3} \dfrac{h(x)}{2k(x)} = \dfrac{4}{2\cdot 5} = \dfrac{2}{5}$

B. $\lim_{x\to 3} \dfrac{\sqrt{h(x) + k(x)}}{x} = \dfrac{\sqrt{4+5}}{3} = \dfrac{3}{3} = 1$

10. (8 points) Use the Intermediate Value Theorem to show that there is a root of $f(x) = 3x^7 + x - 2$ in the interval $(0, 1)$. (**Make sure to mention any properties of $f(x)$ required to apply the Intermediate Value Theorem.**)

$f(x)$ is continuous everywhere.

$f(0) = 3\cdot(0)^7 + 0 - 2 = -2$

$f(1) = 3\cdot 1^7 + 1 - 2 = 2$

By the Intermediate Value Theorem, there exists a c in $(0,1)$ with $f(c) = 0$.

11. (10 points) Find the horizontal asymptote(s) for $y = \dfrac{\sqrt{25x^2+4}}{3x-6}$. (Show your work using limits.)

$$\lim_{x\to\infty} \frac{\sqrt{25x^2+4}}{3x-6} = \lim_{x\to\infty} \frac{\frac{\sqrt{25x^2+4}}{x}}{\frac{3x-6}{x}} = \lim_{x\to\infty} \frac{\frac{\sqrt{25x^2+4}}{\sqrt{x^2}}}{3-\frac{6}{x}}$$

$\boxed{\text{for } x>0 \quad x=\sqrt{x^2}}$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{25x^2+4}{x^2}}}{3-\frac{6}{x}} = \lim_{x\to\infty} \frac{\sqrt{25+\frac{4}{x^2}}}{3-\frac{6}{x}}$$

$$= \frac{\sqrt{25+0}}{3-0} = \frac{5}{3}$$

$$\lim_{x\to-\infty} \frac{\sqrt{25x^2+4}}{3x-6} = \lim_{x\to-\infty} \frac{\frac{\sqrt{25x^2+4}}{x}}{\frac{3x-6}{x}} = \lim_{x\to-\infty} \frac{\frac{\sqrt{25x^2+4}}{-\sqrt{x^2}}}{3-\frac{6}{x}}$$

$\boxed{\text{for } x<0 \quad x=-\sqrt{x^2}}$

$$= \lim_{x\to-\infty} \frac{-\sqrt{\frac{25x^2+4}{x^2}}}{3-\frac{6}{x}} = \lim_{x\to-\infty} \frac{-\sqrt{25+\frac{4}{x^2}}}{3-\frac{6}{x}}$$

$$= \frac{-\sqrt{25+0}}{3-0} = -\frac{5}{3}$$

Horizontal Asymptotes: $y = \frac{5}{3}$ and $y = -\frac{5}{3}$