Name Solutions Rec. Instr.

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Math 220

Exam 2

March 3, 2016

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		36	6		3
2		9	7		7
3		9	8		10
4		9	9		4
5		9	10		4

Total Score:

1. (6 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\dfrac{d}{dx}\left(x^5 + \dfrac{4}{x} + e^3\right)$ $= 5x^4 - \dfrac{4}{x^2}$

B. $\dfrac{d}{dx}(\sin(x) \cdot 2^x)$ $= \cos(x) \cdot 2^x + \sin(x) \cdot 2^x \cdot \ln(2)$

C. $\dfrac{d}{dx}\left(\dfrac{\sqrt{x}}{\cos(x)}\right)$ $= \dfrac{\frac{1}{2\sqrt{x}} \cdot \cos(x) - \sqrt{x} \cdot (-\sin(x))}{\cos^2(x)}$

D. $\dfrac{d}{dw}\arctan(5w^2 + 3)$ $= \dfrac{1}{1 + (5w^2+3)^2} \cdot 10w$

E. $\dfrac{d}{dx}\left(e^{e^x}\right)$ $= e^{e^x} \cdot e^x$

F. $\dfrac{d}{d\theta}(\tan(\theta) \cdot \ln(\theta))$ $= \sec^2(\theta) \cdot \ln(\theta) + \tan(\theta) \cdot \dfrac{1}{\theta}$

2. (9 points) Let $g(x) = x^2 + 3x$. Using the **limit definition of the derivative**, find $g'(2)$. Make sure to use limit notation correctly.

$$g'(2) = \lim_{h \to 0} \frac{g(2+h) - g(2)}{h}$$

$$= \lim_{h \to 0} \frac{(2+h)^2 + 3\cdot(2+h) - (2^2 + 3\cdot 2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} + 4h + h^2 + \cancel{6} + 3h - \cancel{4} - \cancel{6}}{h}$$

$$= \lim_{h \to 0} \frac{7h + h^2}{h}$$

$$= \lim_{h \to 0} (7 + h)$$

$$= 7 + 0$$

$$= 7$$

3. (9 points) Find the equation of the tangent line to the curve $y = \sin(3x) + 7$ at $x = 0$.

$$\frac{dy}{dx} = \cos(3x)\cdot 3 \quad \text{so} \quad \left.\frac{dy}{dx}\right|_{x=0} = \cos(3\cdot 0)\cdot 3 = 3$$

The tangent line to the curve at $x=0$ has slope 3 and goes through $(0, \sin(3\cdot 0)+7) = (0,7)$.

$$y - 7 = 3(x - 0) \quad \text{or} \quad y = 3x + 7$$

4. (9 points) Find $\dfrac{dy}{dx}$ if $x^2y^3 = e^x - y^2$.

$$\frac{d}{dx}[x^2y^3] = \frac{d}{dx}[e^x - y^2]$$

$$2xy^3 + x^2 \cdot 3y^2\frac{dy}{dx} = e^x - 2y\frac{dy}{dx}$$

$$3x^2y^2\frac{dy}{dx} + 2y\frac{dy}{dx} = e^x - 2xy^3$$

$$(3x^2y^2 + 2y)\frac{dy}{dx} = e^x - 2xy^3$$

$$\frac{dy}{dx} = \frac{e^x - 2xy^3}{3x^2y^2 + 2y}$$

5. (9 points) Find the derivative of $h(x) = \dfrac{x^x}{(3x^2+4)^5}$.

$$\ln(h(x)) = \ln\left(\frac{x^x}{(3x^2+4)^5}\right) = \ln(x^x) - \ln((3x^2+4)^5)$$

$$= x \cdot \ln(x) - 5 \cdot \ln(3x^2+4)$$

$$\frac{d}{dx}\ln(h(x)) = \frac{d}{dx}[x \cdot \ln(x) - 5 \cdot \ln(3x^2+4)]$$

$$\frac{h'(x)}{h(x)} = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 5 \cdot \frac{6x}{3x^2+4}$$

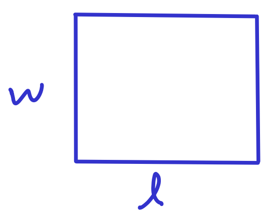
$$h'(x) = h(x) \cdot \left(\ln(x) + 1 - \frac{30x}{3x^2+4}\right)$$

$$h'(x) = \frac{x^x}{(3x^2+4)^5}\left(\ln(x) + 1 - \frac{30x}{3x^2+4}\right)$$

6. (3 points) Suppose that a waiter brings you a bowl of boiling soup. Let $F(t)$ denote the temperature in degrees Fahrenheit of the soup after t minutes. Is $F'(2)$ positive or negative? Explain your answer.

$F'(2)$ should be negative because the temperature should be decreasing 2 minutes after the hot soup arrives.

7. (7 points) The length of a rectangle is increasing at a rate of 2 ft/s, and its width is increasing at a rate of 3 ft/s. At what rate is the area of the rectangle increasing when the length is 4 ft and the width is 5 ft?



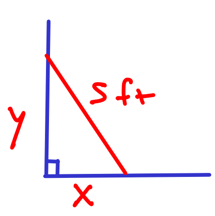
Area: $A = lw$

Given: $\frac{dl}{dt} = 2$ ft/s, $\frac{dw}{dt} = 3$ ft/s

Want: $\frac{dA}{dt}$ when $l = 4$ ft and $w = 5$ ft.

$\frac{dA}{dt} = \frac{dl}{dt} \cdot w + l \cdot \frac{dw}{dt}$ so when $l = 4$ ft, $w = 5$ ft, we have $\frac{dA}{dt} = 2 \cdot 5 + 4 \cdot 3 = 22$ ft²/s.

8. (10 points) A 5-foot ladder rests against the wall. If the top of the ladder slides down the wall at a rate of 3 ft/s, how fast is the bottom of the ladder moving away from the wall when the top of the ladder is 4 feet above the ground?



Given: $\frac{dy}{dt} = -3$ ft/s

Want: $\frac{dx}{dt}$ when $y = 4$ ft

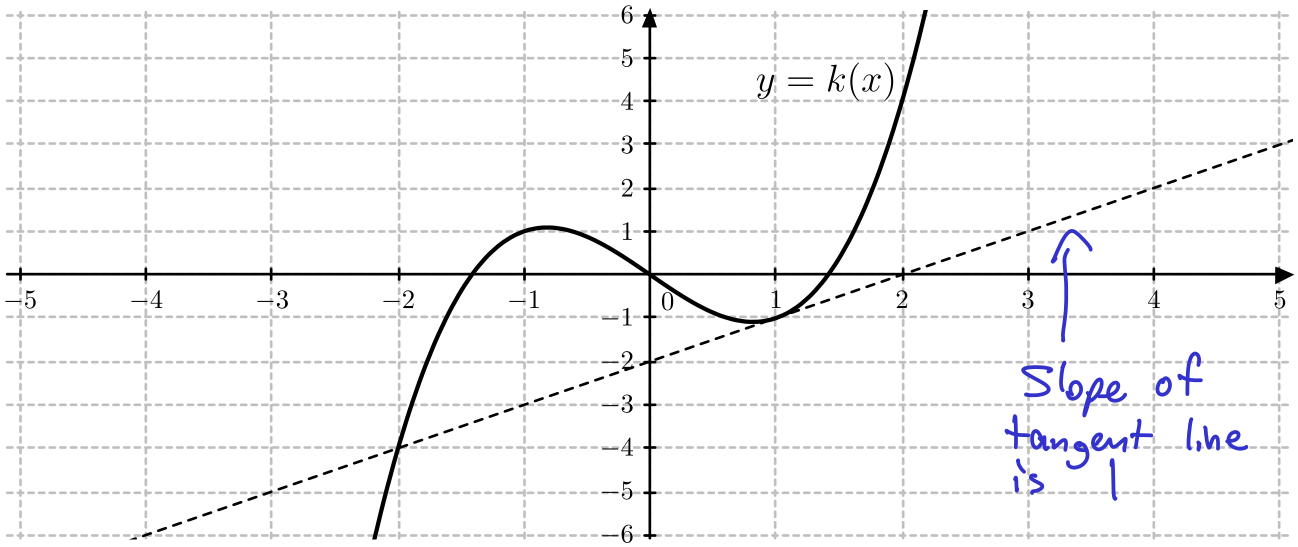
$x^2 + y^2 = 5^2$

$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt} 5^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ so $\frac{dx}{dt} = -\frac{y}{x} \cdot \frac{dy}{dt}$

When $y = 4$ ft, $x = \sqrt{5^2 - 4^2} = 3$ ft, and

$\frac{dx}{dt} = -\frac{4}{3} \cdot (-3) = 4$ ft/s



9. (2 points each) The function $y = k(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values.

A. $k(1) = -1$

B. $k'(1) = 1$

x	$v(x)$	$w(x)$	$v'(x)$	$w'(x)$
1	4.5	0	1	2.5
2	5	2	3	4
3	12	9	8	-1
4	21	8	10	-2

10. (4 points) Given that $f(x) = \dfrac{v(x)}{w(x)}$, use the table above to calculate $f'(2)$.

$$f'(x) = \frac{v'(x)w(x) - v(x)\cdot w'(x)}{(w(x))^2}$$

$$f'(2) = \frac{v'(2)w(2) - v(2)\cdot w'(2)}{(w(2))^2}$$

$$= \frac{3\cdot 2 - 5\cdot 4}{2^2} = \frac{-14}{4} = -\frac{7}{2}$$