Name Solutions Rec. Instr.

Signature Rec. Time

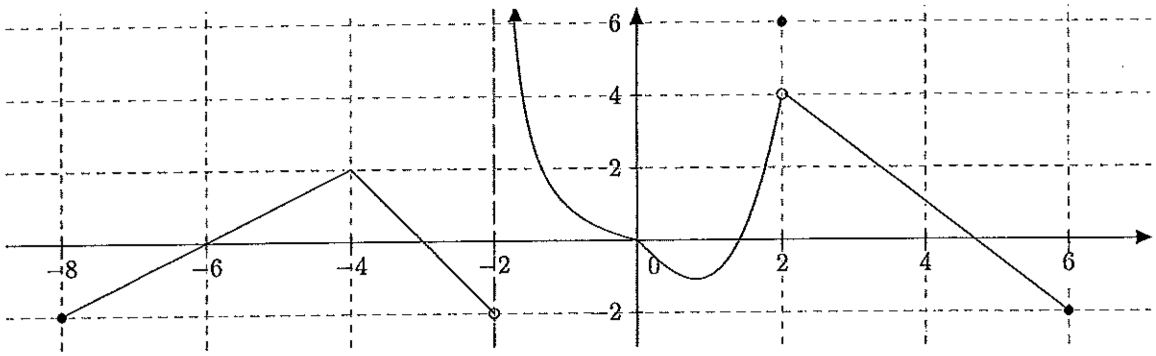
Math 220

Exam 1

September 21, 2017

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		14	6		8
2		10	7		6
3		5	8		18
4		15	9		8
5		6	10		10
			Total Score		100



1. (2 points each) Consider the graph of $y = f(x)$ above. State the value of each of the below quantities. If the limit or derivative does not exist, write "does not exist", or $\pm\infty$ if appropriate. No work needs to be shown.

A. $\lim_{x \to -2^-} f(x) =$ -2

B. $\lim_{x \to -2^+} f(x) =$ ∞

C. $\lim_{x \to 2} f(x) =$ 4

D. $\lim_{x \to -4} f(x) =$ 2

E. $f'(3.7) =$ $\frac{\Delta y}{\Delta x} = \frac{-6}{4} = -3/2$ ← slope of line segment

F. Give all removable discontinuities of $f(x)$: $x =$ 2

G. Give all points where $f(x)$ is continuous but not differentiable: $x =$ $-4, 0$

Not clear from graph, so full credit is given whether you include 0 or not.

(It is also acceptable to include the endpoints $-8, 6$)

2. (5 points each) An object moves along a straight line with position s meters at time t seconds. Given the table of data below answer the following questions.

t	2	2.1	3	4	5
s	1	2	5	10	16

A. Determine the average velocity of the object over the time interval $2 \leq t \leq 5$ seconds.

$$V_{ave} = \frac{\Delta s}{\Delta t} = \frac{s(5)-s(2)}{5-2} = \frac{16-1}{3} = 5 \text{ m/sec.}$$

B. Give the best estimate of the instantaneous velocity of the object at time $t = 2$ seconds, based on the given data.

$$v(2) \approx \frac{s(2.1)-s(2)}{2.1-2} = \frac{2-1}{.1} = \frac{1}{1/10} = 10 \text{ m/sec.}$$

3. (5 points) Let $f(x) = x - 3x^3 + 1$. Use the Intermediate Value Theorem to show that there is a point c between 0 and 1 such that $f(c) = 0$. Give a careful explanation to receive full credit.

$f(0) = 0+0+1 = 1$, $f(1) = 1-3+1 = -1$

$f(x)$ is continuous on $[0, 1]$.

Since 0 is between 1 and -1, by IVT, there is a point c with $f(c) = 0$ and $0 \leq c \leq 1$.

4. (5 points each) Evaluate the following limits. Write ∞ or $-\infty$ for infinite limits, and "does not exist" for limits that do not exist.

A. $\lim_{x\to 2} \dfrac{x^2+x-6}{x-2}$ $= \lim_{x\to 2} \dfrac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}} = 2+3 = 5$

B. $\lim_{t\to 2^+} \dfrac{1-t}{t^2-4}$ $= \lim_{t\to 2^+} \dfrac{1-t}{(t+2)(t-2)}$ Let $t \approx 2.01$

$\approx \dfrac{1-2}{(2+2)(2.01-2)} = \dfrac{-1}{4\cdot(.01)} = \dfrac{-1}{4}\cdot 100$

$\to -\infty$ or $= -\infty$

C. $\lim_{h\to 9} \dfrac{h-9}{\sqrt{h}-3} \cdot \dfrac{(\sqrt{h}+3)}{(\sqrt{h}+3)} = \lim_{h\to 9} \dfrac{\cancel{(h-9)}(\sqrt{h}+3)}{\cancel{(h-9)}}$

$= \sqrt{9}+3 = 6$

5. (6 points) Let $f(x) = \begin{cases} 3x - c & \text{if } x < 2 \\ x^2 - x + 7 & \text{if } x \geq 2 \end{cases}$

A. Find $\lim_{x \to 2^+} f(x)$

$$= \lim_{x \to 2^+} x^2 - x + 7 = 4 - 2 + 7 = 9$$

B. Find the value of c so that $f(x)$ is continuous on $(-\infty, \infty)$.

$$3 \cdot 2 - c = 9 \quad \Rightarrow \quad c = 6 - 9 = -3$$

6. (8 points) Use the limit definition of derivative to find $f'(x)$ where $f(x) = 2x - x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h) - (x+h)^2 - (2x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} + 2h - (\cancel{x^2} + 2xh + h^2) - \cancel{2x} + \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{2h - 2xh - h^2}{h} = \lim_{h \to 0} 2 - 2x - h$$

$$= 2 - 2x$$

7. (6 points) Calculate the derivative of the following function using rules for derivatives, not the limit definition of the derivative.

$w(x) = \frac{1}{x^3} - \sqrt[3]{x} + \pi = x^{-3} - x^{1/3} + \pi$

$w'(x) = -3x^{-4} - \frac{1}{3}x^{-2/3}$

8. Calculate the derivative of the following functions using rules for derivatives, not the limit definition of the derivative. You do not need to simplify your answer.

A. (6 points) $v(x) = \sin^3(x^7 - \cos(x))$ $= (\sin(x^7 - \cos x))^3$

$v'(x) = 3\sin^2(x^7 - \cos x) \cdot \cos(x^7 - \cos x) \cdot (7x^6 + \sin x)$

B. (6 points) $h(x) = (2x+1)^3 \sec(x)$

$h'(x) = (2x+1)^3 \sec x \tan x + \sec x \cdot 3(2x+1)^2 \cdot 2$

C. (6 points) $g(x) = \frac{x^2-1}{7x-3}$

$$g'(x) = \frac{(7x-3)\,2x - (x^2-1)\cdot 7}{(7x-3)^2}$$

9. (8 points) Find the equation of the tangent line to the curve $y = 4 - x^3$ at the point where $x = 1$.

$dy/dx = -3x^2$, $\left. dy/dx \right|_{x=1} = -3$

Point: $x = 1$, $y = 4 - 1 = 3$

Pt-Slope: $y - y_0 = m(x - x_0)$

$y - 3 = -3(x - 1)$

or $y = -3x + 6$

10. An object starting at $t = 0$ is traveling along a horizontal line with position function $s(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 - 2t$, with s measured in feet and t in seconds.

A. (3 points) Find the velocity of the object at time t.

$v(t) = s'(t) = \frac{1}{3}3t^2 - \frac{1}{2}2t - 2 = t^2 - t - 2$

$= (t-2)(t+1)$

B. (3 points) Find the acceleration of the object at time t seconds.

$a(t) = v'(t) = 2t - 1$

$-$ $+$

0 $1/2$

C. (2 points) Determine the time interval (with $t \geq 0$) when the object is moving to the right, that is, in the direction of increasing s.

v $-$ $+$

0 2 test $(t-2)(t+1)$

$v > 0$ when $t > 2$ or $(2, \infty)$

D. (2 points) Determine the time interval(s) (with $t \geq 0$) when the object is slowing down, that is, its speed is decreasing.

Speed decreases when v and a have opposite signs.

$\frac{1}{2} < t < 2$ seconds, where $a > 0$ and $v < 0$.

or $(\frac{1}{2}, 2)$