Name Solutions Rec. Instr.
Signature Rec. Time

Math 220
Exam 3
November 16, 2017

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. **Show your work.**

Problem	Points	Points Possible	Problem	Points	Points Possible
1		18	6		6
2		18	7		7
3		10	8		10
4		8	9		6
5		7	10		10

Total Score:

1. Evaluate the following indefinite integrals.

A. (6 points) $\displaystyle\int \frac{\sqrt{x}-\sqrt{2}\,x^5}{x}\,dx$

$= \int x^{-1/2} - \sqrt{2}\,x^4\,dx = 2x^{1/2} - \sqrt{2}\,\frac{x^5}{5} + C$

B. (6 points) $\displaystyle\int 3x\sin(5x^2)\,dx$

$u = 5x^2$

$du = 10x\,dx \Rightarrow x\,dx = \frac{1}{10}\,du$

$= 3\int \sin(5x^2)\,x\,dx$

$= 3\int \sin(u)\,\frac{1}{10}\,du = \frac{3}{10}(-)\cos(u) + C$

$= -\frac{3}{10}\cos(5x^2) + C$

C. (6 points) $\displaystyle\int x\sqrt{x+2}\,dx$

$u = x+2, \quad x = u-2$

$du = dx$

$= \int (u-2)\sqrt{u}\,du$

$= \int u^{3/2} - 2u^{1/2}\,du = \frac{2}{5}u^{5/2} - 2\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

2. Evaluate the following limits.

A. (6 points) $\lim_{t\to 0} \frac{1+t-\cos(t)}{t^2+\sin(2t)}$ $\frac{1+0-1}{0+0} = \frac{0}{0}$ - type

by L'Hopital $= \lim_{t\to 0} \frac{1+\sin(t)}{2t+2\cos(2t)}$

$= \frac{1+\sin(0)}{0+2\cos(0)} = \frac{1+0}{0+2} = \frac{1}{2}$

B. (6 points) $\lim_{x\to\infty} x^2 2^{-x}$ $= \lim_{x\to\infty} \frac{x^2}{2^x}$ $\frac{\infty}{\infty}$ - type

by L'Hopital $= \lim_{x\to\infty} \frac{2x}{2^x \ln 2}$ $\frac{\infty}{\infty}$ - type

$= \lim_{x\to\infty} \frac{2}{2^x(\ln 2)^2} = \frac{2}{\infty} = 0$

C. (6 points) $\lim_{x\to 0^+} x^{2x}$ $= L$ $0\cdot(-\infty)$

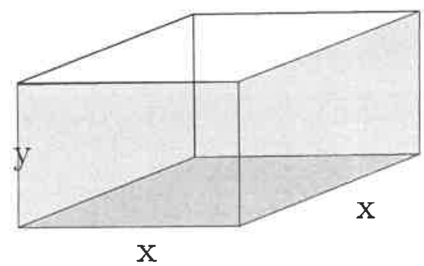
$\ln L = \lim_{x\to 0^+} \ln(x^{2x}) = \lim_{x\to 0^+} 2x\ln x$

$= \lim_{x\to 0^+} \frac{2\ln x}{1/x}$ $\frac{\infty}{\infty}$ - type

by L'Hopital $= \lim_{x\to 0^+} \frac{2\cdot 1/x}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{2}{x}\cdot\frac{-x^2}{1} = \lim_{x\to 0^+} -2x$
$= 0$

Thus $\ln L = 0 \Rightarrow L = e^0 = \boxed{1}$

3. (10 points) A box with square base and open top is formed from 2 materials. The base costs \$4 per square foot, while the four sides cost \$1 per square foot. If the total cost for the base and four sides is fixed to be \$120, find the dimensions that maximize the volume of the box.



$C = \text{cost} = 4x^2 + 4xy = 120$

Maximize $V = \text{volume} = x^2 y$

$$x^2 + xy = \tfrac{1}{4} \cdot 120 = 30$$

$$\Rightarrow xy = 30 - x^2 \Rightarrow y = \frac{30 - x^2}{x}$$

Thus $V = x^2 y = x^2 \cdot \left(\frac{30 - x^2}{x}\right) = x(30 - x^2)$

$$V = 30x - x^3$$

$$\frac{dV}{dx} = 30 - 3x^2 = 0 \Rightarrow 3x^2 = 30, \quad x^2 = 10$$

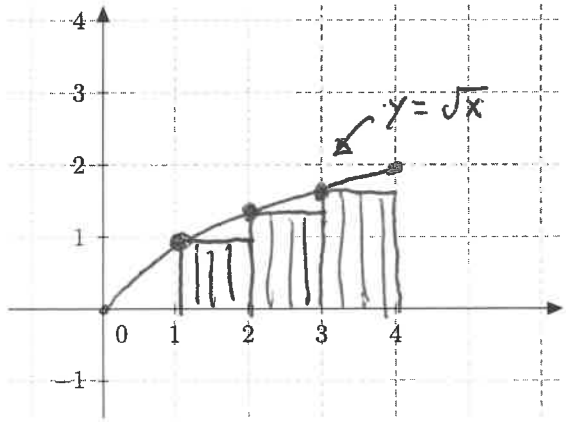
$x = \sqrt{10}$ ft (need $x > 0$)

$y = \frac{30 - x^2}{x} = \frac{20}{\sqrt{10}} = 2\sqrt{10}$ ft

To justify that this is a maximum we examine the sign of dV/dx

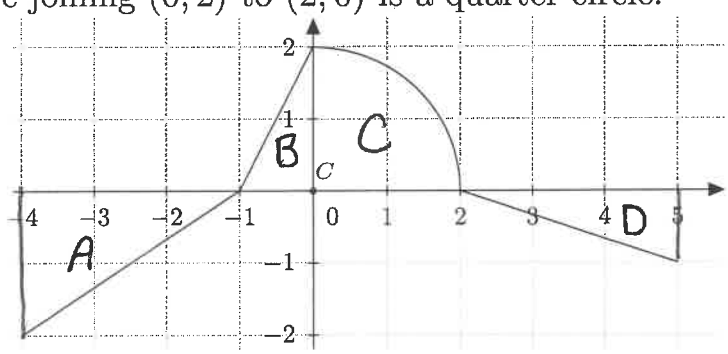
$\frac{dV}{dx}$: $+$ | $-$ at $\sqrt{10}$

4. (8 points) Estimate the area below the curve $y = \sqrt{x}$ over the interval [1,4] using L_3, the left end point approximation with three intervals. Also, make a sketch of the graph of $y = \sqrt{x}$ and illustrate the rectangles on your graph. (Leave your answer as a sum. Do not simplify. $\sqrt{2} \approx 1.4$, $\sqrt{3} \approx 1.7$.)



$$L_3 = 1 \cdot 1 + \sqrt{2} \cdot 1 + \sqrt{3} \cdot 1$$
$$= 1 + \sqrt{2} + \sqrt{3}$$

5. (7 points) For the function $f(x)$ graphed below evaluate the given integral. The arc joining $(0,2)$ to $(2,0)$ is a quarter-circle.



$$\int_{-4}^{5} f(x)\,dx = -A + B + C - D$$
$$= -\tfrac{1}{2}\,3 \cdot 2 + \tfrac{1}{2}\,1 \cdot 2 + \tfrac{1}{4}\pi\, 2^2 - \tfrac{1}{2}\,3 \cdot 1$$
$$= -3 + 1 + \pi - 3/2$$
$$= -\frac{7}{2} + \pi$$

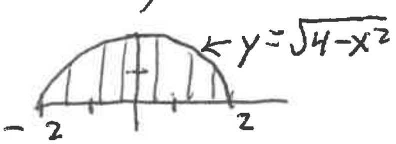
6. (6 points) Evaluate the following integral. Just use symmetry and geometry.

$$\int_{-2}^{2} \sin^3(5x) + \sqrt{4-x^2}\,dx$$

odd since $\sin^3(-5x) = -\sin^3(5x)$, so $\int_{-2}^{2} \sin^3(5x)\,dx = 0$

$= \int_{-2}^{2} \sqrt{4-x^2}\,dx$ = area of semi-circle of radius 2

$= \frac{1}{2}\pi \cdot 2^2 = \boxed{2\pi}$



7. (7 points) Find the average value of $f(x) = \frac{1}{x}$ over the interval $[1, e]$. (Here, e is the natural logarithm base.)

$$f_{ave} = \frac{1}{e-1} \int_1^e \frac{1}{x}\,dx$$

$$= \frac{1}{e-1} \ln x \Big|_1^e = \frac{1}{e-1}(\overset{1}{\ln e} - \overset{0}{\ln 1})$$

$$= \boxed{\frac{1}{e-1}}$$

8. (10 points) Solve the initial value problem for $f(t)$: $f'(t) = 2e^{-2t}$, $f(0) = 1$.

$$f(t) = \int 2e^{-2t}\,dt = \cancel{2}\,e^{-2t} \cdot \frac{1}{\cancel{-2}} + C = -e^{-2t} + C$$

$$f(0) = 1 \Rightarrow -e^{-2\cdot 0} + C = 1$$

$$\Rightarrow -1 + C = 1 \Rightarrow C = 2$$

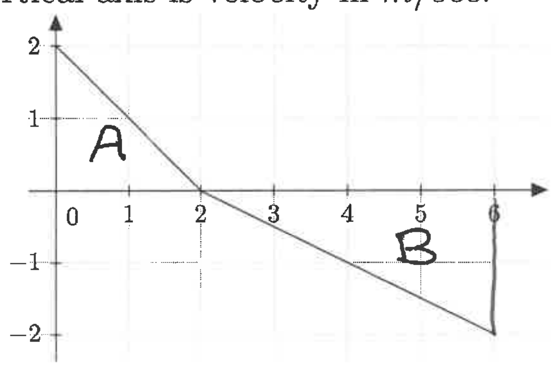
$$\boxed{f(t) = -e^{-2t} + 2}$$

9. (3 points) **A.** Find $\dfrac{d}{dx}\displaystyle\int_2^x \frac{\cos(t^2)}{2+t}\,dt$ $= \dfrac{\cos(x^2)}{2+x}$ by Fund. Theorem of Calc.

(3 points) **B.** Find $\dfrac{d}{dx}\displaystyle\int_{x^3}^5 \frac{\cos(t^2)}{2+t}\,dt$ $= \dfrac{d}{dx}(-)\displaystyle\int_5^{x^3} \frac{\cos(t^2)}{2+t}\,dt$

$= -\dfrac{\cos(x^6)}{2+x^3}\cdot 3x^2$, by F.T.C. and chain rule

10. The velocity function $v = v(t)$ for an object moving along a straight line is graphed below. The horizontal axis is time measured in seconds, and the vertical axis is velocity in m/sec.



a. (5 points) Let $s = s(t)$ denote the position of the object. If the object is at position $s = 3$ when $t = 0$, where is it after 6 seconds?

$s(6) - s(0) = \displaystyle\int_0^6 v(t)\,dt = A - B = \tfrac{1}{2}\cdot 2\cdot 2 - \tfrac{1}{2}\,4\cdot 2 = 2 - 4$

$s(6) - s(0) = -2$

Thus $s(6) = s(0) - 2 = 3 - 2 = \boxed{1\ m}$

b. (5 points) Find the total distance the object travels during the time interval $[0, 6]$ seconds.

Total dist $= \displaystyle\int_0^6 |v(t)|\,dt = A + B = 2 + 4 = \boxed{6\ m}$