Name ______________________ Rec. Instr. ______________________
Signature ______________________ Rec. Time ______________________

Math 220
Final Exam
December 13, 2017

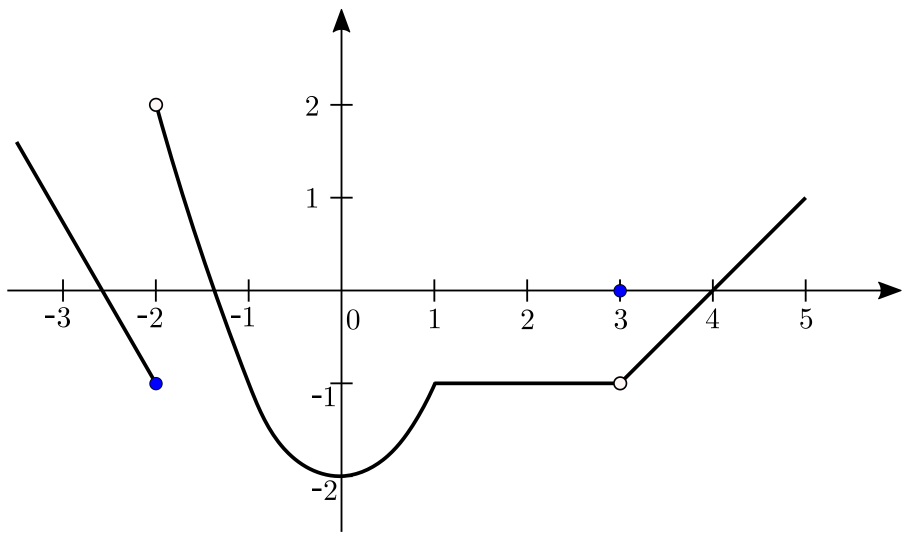
No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 1 hour and 50 minutes to complete the exam.

Total = 200 points. **Show your work unless stated otherwise.**

Problem	Points	Points Possible	Problem	Points	Points Possible
1		12	9		10
2		18	10		10
3		8	11		8
4		24	12		24
5		12	13		12
6		10	14		8
7		10	15		8
8		10	16		16

Total Score:

1. (2 points each) Evaluate the following for the function $f(x)$ graphed below or state that it does not exist. No work needs to be shown.



a. $\lim_{x \to -2^+} f(x) =$

b. $\lim_{x \to 1} f(x) =$

c. $\lim_{x \to 3} f(x) =$

d. $f'(3.5) =$

e. Indicate all values of x between -3 and 4 at which $f(x)$ is not continuous.

f. Indicate all values of x between -3 and 4 at which $f'(x)$ is not defined.

2. (6 points each) Evaluate the following limits.

a. $\lim_{x \to 2} \dfrac{x^3 - 4x}{2 - x} =$

b. $\lim_{h \to 0} \dfrac{\sin(3h)}{\sin(2h)} + \dfrac{\cos(3h)}{\cos(2h)} =$

c. $\lim_{x\to\infty} (x^2+5)^{1/x}$

3. (8 points) Use the definition of derivative as a limit to find $f'(x)$ for $f(x) = x^2 + x$.

4. (8 points each) Compute the following derivatives. **DO NOT SIMPLIFY**

a. $f'(t)$ where $f(t) = \sin^5(\ln t)$

b. $\dfrac{d}{dx}\, e^{3x} \tan^{-1}(x)$ (Here, $\tan^{-1}(x) = \arctan(x)$.)

c. $\dfrac{d}{dx}\, \dfrac{x + \tan x}{1 - x^2} =$

5. (4 points each) Let $f(x)$ be a function with $f'(x) = x^2(x^2 - 4)(x - 7)$.
a. Find the critical points of $f(x)$.

b. Find the open intervals where $f(x)$ is increasing and decreasing.

c. Classify each critical point as a local minimum, local maximum or neither.

6. Let $g(x) = 2x^6 - 5x^4$.

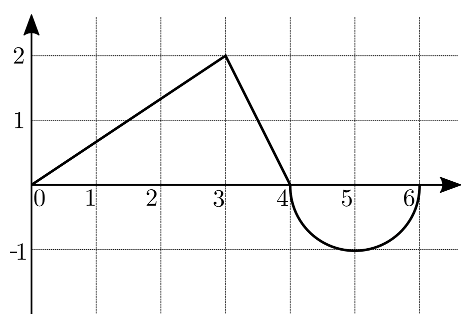
a. (6 points) Determine the open intervals where $g(x)$ is concave up and concave down.

b. (4 points) Determine all inflection points of $g(x)$. Just give the x-coordinates.

7. (10 points) Consider a rectangle with edges of length x, y. If x is increasing at a rate of 5 m/sec and y is decreasing at a rate of 2 m/sec, at what rate is the area A of the rectangle changing when $x = 3$ m and $y = 4$ m?

8. (10 points) Use implicit differentiation to find the equation of the tangent line to the curve $xy + y^2 = 2x - 1$ at $(2, 1)$.

9. The velocity function $v = v(t)$ for an object moving along a straight line is graphed below. The horizontal axis is time measured in seconds, and the vertical axis is velocity in m/sec. The arc from (4,0) to (6,0) is a semicircle.



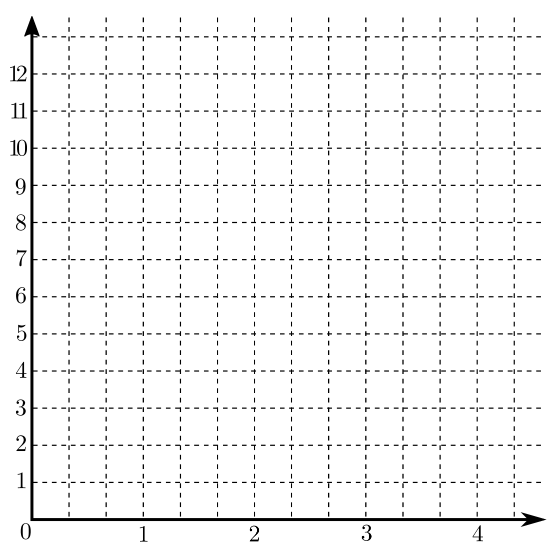
a. (2 points) State the time intervals when the object is moving to the right, and when it is moving to the left.

b. (4 points) Let $s = s(t)$ denote the position of the object. If the object is at position $s = -3$ when $t = 0$, where is it after 6 seconds?

c. (4 points) Find the total distance the object travels during the time interval $[0, 6]$ seconds.

10. (10 points) A rectangular fence consists of three sides costing \$2 per meter, and one side costing \$1 per meter. If the area of the rectangle is 12 square meters, find the dimensions that minimize the cost of the fence.

11. (8 points) Estimate the area below the curve $y = x^2 + 2$ over the interval [0,3] using R_3, the right end point approximation with three rectangles. Also, make a sketch of the graph of $y = x^2 + 2$ and illustrate the rectangles on your graph.



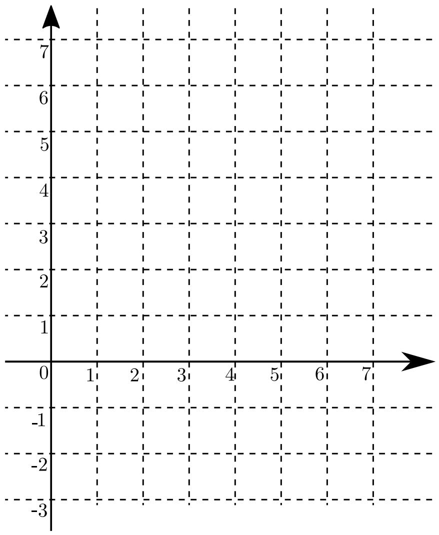
12. (8 points each) Evaluate the following integrals.

a. $\displaystyle\int e^{5x} - \frac{1}{\sqrt{4-x^2}}\, dx$

b. $\displaystyle\int \sin^5(2x)\cos(2x)\, dx$

c. $\displaystyle\int_1^e \frac{(\ln x)^2}{x}\, dx$

13. (12 points) Make a sketch of the region with $x \geq 0$ bounded by the y-axis, the parabola $y = x^2 - 2x$ and the line $y = 6 - x$, and then calculate its area.

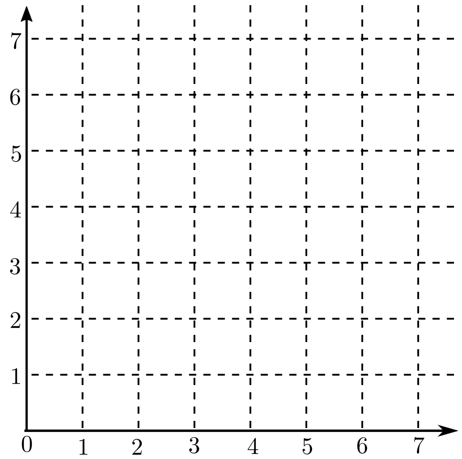


14. (8 points) Solve the initial value problem: $f'(t) = 4t^3 - \sin t$, $f(0) = 1$.

15. (8 points) a) Find the linear approximation of $f(x) = \sqrt{x}$ near $x = 9$.

b) Use your estimate in part a) to estimate $\sqrt{8.9}$.

16. a. (4 points) Sketch the region bounded by the y-axis and the lines $y = x$, $y = 3 + \frac{1}{2}x$, and find the indicated volumes.



b. (6 points) The volume of the solid obtained by rotating the region around the y-axis. Just set up the integral. You do not need to evaluate it.

c. (6 points) The volume of the solid obtained by rotating the region around the x-axis. Just set up the integral. You do not need to evaluate it.