Name Solutions ______________________ Rec. Instr. ______________________

Signature ______________________ Rec. Time ______________________

Math 220

Exam 2

March 2, 2017

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		30	6		10
2		6	7		10
3		10	8		5
4		10	9		9
5		10	**Total Score**		100

1. (6 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\dfrac{d}{dx}\left(2^x - \dfrac{5}{x^2} + \ln(7)\right)$ $= 2^x \cdot \ln(2) + \dfrac{10}{x^3}$

B. $\dfrac{d}{dx}\left(\sqrt{x} \cdot \tan(x)\right)$ $= \dfrac{1}{2\sqrt{x}} \cdot \tan(x) + \sqrt{x} \cdot \sec^2(x)$

C. $\dfrac{d}{d\theta}\cos\left(\sin(5\theta)\right)$ $= -\sin(\sin(5\theta)) \cdot \left(\dfrac{d}{d\theta}\sin(5\theta)\right)$

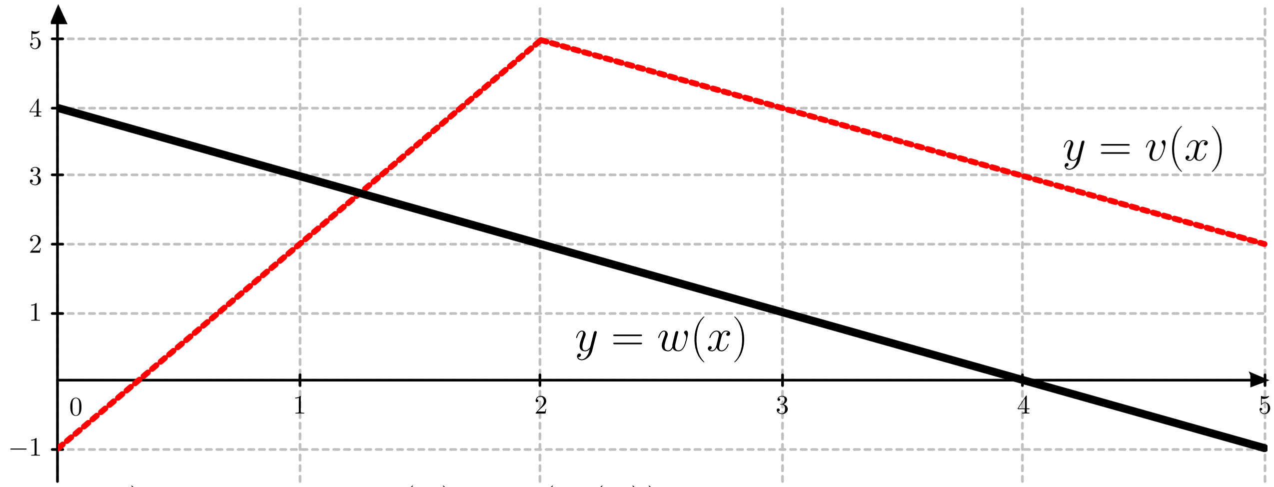
$= -\sin(\sin(5\theta)) \cdot \cos(5\theta) \cdot \left(\dfrac{d}{d\theta} 5\theta\right)$

$= -\sin(\sin(5\theta)) \cdot \cos(5\theta) \cdot 5$

D. $\dfrac{d}{dt}\arctan(2t^2 - 3)$ $= \dfrac{1}{1+(2t^2-3)^2}\left(\dfrac{d}{dt}(2t^2-3)\right)$

$= \dfrac{4t}{1+(2t^2-3)^2}$

E. $\dfrac{d}{dx}\left(\dfrac{6\ln(x) - 2x^3}{e^x + 3}\right)$ $= \dfrac{\left(\frac{6}{x} - 6x^2\right)(e^x+3) - (6\ln(x) - 2x^3) \cdot e^x}{(e^x+3)^2}$



2. (6 points) Given that $f(x) = v(w(x))$, use the graph above to calculate $f'(3)$.

$$f'(x) = v'(w(x)) \cdot w'(x)$$
$$f'(3) = v'(w(3)) \cdot w'(3)$$
$$= v'(1) \cdot (-1)$$
$$= -3$$

3. (10 points) Find the derivative of $h(x) = e^x \cdot x^{5\cos(x)}$.

$$\ln(h(x)) = \ln(e^x \cdot x^{5\cos(x)}) = \ln(e^x) + \ln(x^{5\cos(x)})$$
$$= x + 5\cos(x) \cdot \ln(x)$$
$$\frac{d}{dx} \ln(h(x)) = \frac{d}{dx}(x + 5\cos(x) \cdot \ln(x))$$
$$\frac{h'(x)}{h(x)} = 1 - 5\sin(x) \cdot \ln(x) + \frac{5\cos(x)}{x}$$
$$h'(x) = h(x)\left(1 - 5\sin(x) \cdot \ln(x) + \frac{5\cos(x)}{x}\right)$$
$$= e^x \cdot x^{5\cos(x)}\left(1 - 5\sin(x)\ln(x) + \frac{5\cos(x)}{x}\right)$$

4. (10 points) Let $g(x) = \dfrac{3}{x}$. Using the **limit definition of the derivative**, find $g'(x)$. Make sure to use limit notation correctly.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$$

$$= \lim_{h\to 0} \left(\frac{3}{(x+h)h} - \frac{3}{xh} \right) = \lim_{h\to 0} \left(\frac{3x}{(x+h)xh} - \frac{3(x+h)}{(x+h)xh} \right)$$

$$= \lim_{h\to 0} \frac{\cancel{3x} - \cancel{3x} - 3h}{(x+h)xh} = \lim_{h\to 0} \frac{-3\cancel{h}}{(x+h)x\cancel{h}}$$

$$= \lim_{h\to 0} \frac{-3}{(x+h)x} = \frac{-3}{(x+0)x} = \frac{-3}{x^2}$$

5. (10 points) Find the equation of the tangent line to the curve $y = 4 + \dfrac{x}{2x+1}$ at $x = 0$.

$$\frac{dy}{dx} = \frac{1\cdot(2x+1) - x\cdot 2}{(2x+1)^2} = \frac{1}{(2x+1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{1}{(2\cdot 0+1)^2} = 1$$

The tangent line has slope 1 and hits the y-axis at $4 + \frac{0}{2\cdot 0+1} = 4$.

$$\boxed{y = x+4}$$

6. (10 points) Find $\dfrac{dy}{dx}$ if $\sin(xy^2) = x^2$.

$$\frac{d}{dx}\sin(xy^2) = \frac{d}{dx}x^2$$

$$\cos(xy^2)\cdot\left(\frac{d}{dx}(xy^2)\right) = 2x$$

$$\cos(xy^2)\left(1\cdot y^2 + x\cdot 2y\frac{dy}{dx}\right) = 2x$$

$$y^2\cos(xy^2) + 2xy\cos(xy^2)\frac{dy}{dx} = 2x$$

$$2xy\cos(xy^2)\frac{dy}{dx} = 2x - y^2\cos(xy^2)$$

$$\frac{dy}{dx} = \frac{2x - y^2\cos(xy^2)}{2xy\cos(xy^2)}$$

7. (10 points) Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and the volume V satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant, the volume is 300 cm^3, the pressure is 100 kPa, and the pressure is increasing at a rate of 20 kPa/min. At what rate is the volume changing at this instant?

Want: $\dfrac{dV}{dt}$ when $V = 300\,\text{cm}^3$, $P = 100$ kPa, and $\dfrac{dP}{dt} = 20$ kPa/min.

$$PV = C$$

$$\frac{d}{dt}PV = \frac{d}{dt}C$$

$$\frac{dP}{dt}\cdot V + P\cdot\frac{dV}{dt} = 0$$

$$P\cdot\frac{dV}{dt} = -\frac{dP}{dt}\cdot V$$

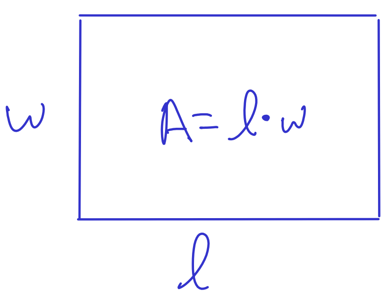
$$\frac{dV}{dt} = -\frac{dP}{dt}\cdot\left(\frac{V}{P}\right)$$

At this instant, $\dfrac{dV}{dt} = -20\cdot\left(\dfrac{300}{100}\right) = -60$ cm^3/min.

8. (5 points) Let $W(t)$ denote the number of km^3 of water in Tuttle Creek Lake t years after January 1, 2000. What does it mean if $W'(18) = -\frac{1}{5}\,\text{km}^3/\text{year}$?

On January 1, 2018, the amount of water in Tuttle Creek Lake is decreasing at a rate of $\frac{1}{5}\ \text{km}^3/\text{year}$.

9. (9 points) The length of a rectangle is increasing at a rate of 2 ft/s, and its width is increasing at a rate of 3 ft/s. At what rate is the area of the rectangle increasing when the length is 6 ft and the width is 7 ft?



Want: $\frac{dA}{dt}$ when $l=6$ ft and $w=7$ ft

Know: $\frac{dl}{dt} = 2$ ft/s and $\frac{dw}{dt} = 3$ ft/s.

$$A = l \cdot w$$

$$\frac{dA}{dt} = \frac{dl}{dt} \cdot w + l \cdot \frac{dw}{dt}$$

When $l = 6$ ft and $w = 7$ ft,

$$\frac{dA}{dt} = 2 \cdot 7 + 6 \cdot 3 = 14 + 18 = 32\ \text{ft}^2/\text{s}$$