Name _____ Rec. Instr._____
Signature _____ Rec. Time_____

Math 220
Exam 3
April 6, 2017

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		16	6		10
2		10	7		10
3		12	8		6
4		6	9		12
5		18	Total Score		100

1. The function $f(x)$ and its first and second derivatives are:

$$f(x) = \frac{x^2}{x^2+3} \qquad f'(x) = \frac{6x}{(x^2+3)^2} \qquad f''(x) = \frac{-18(x^2-1)}{(x^2+3)^3}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: _____

B. (1 point) y-intercept: _____

C. (1 point) x-intercept(s): _____

D. (1 point) Horizontal asymptote(s): _____

E. (1 point) Interval(s) $f(x)$ is increasing: _____

F. (1 point) Interval(s) $f(x)$ is decreasing: _____

G. (1 point) Local maximum(s) (x, y): _____

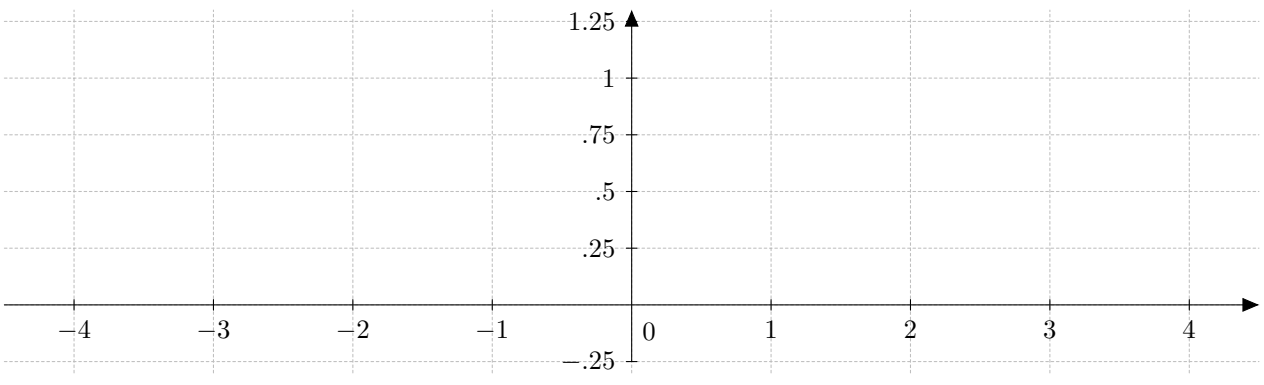
H. (1 point) Local minimum(s) (x, y): _____

I. (1 point) Interval(s) $f(x)$ is concave up: _____

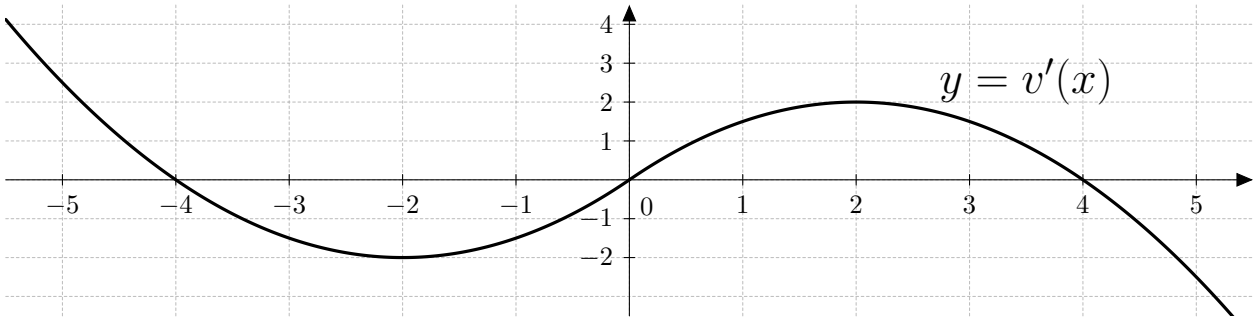
J. (1 point) Interval(s) $f(x)$ is concave down: _____

K. (1 point) Inflection point(s) (x, y): _____

L. (5 points) Sketch $y = f(x)$ on the graph below.



2. (10 points) Find the absolute maximum and absolute minimum of $w(x) = 2x^3 - 9x^2 + 3$ on $[-1, 1]$.



3. (3 points each) $y = v'(x)$ is plotted above. Find:

A. Interval(s) where $v(x)$ is increasing: _____ decreasing: _____

B. x-coordinate(s) where $v(x)$ has a local max: _____ local min: _____

C. Interval(s) where $v(x)$ is concave up: _____ concave down: _____

D. x-coordinate(s) where $v(x)$ has an inflection point: _____

4. (3 points each) In each of the following blanks, fill in "**max**" or "**min**".

A. If $h'(3) = 0$ and $h''(3) = -52$, then $h(x)$ has a local _____ at $x = 3$.

B. If $h'(-2) = 0$ and $h''(-2) = 37$, then $h(x)$ has a local _____ at $x = -2$.

5. (6 points each) Find the following limits. (Use limit notation correctly.)

A. $\lim_{x \to \infty} \dfrac{e^x + 5x}{x + 3}$

B. $\lim_{\theta \to 0} \dfrac{\sin(\theta^2)}{3\theta^2}$

C. $\lim_{x \to -\infty} \dfrac{3x + 5}{\sqrt{4x^2 + 7x}}$

6. (10 points) Let $p(x) = 100 - 2x$ be the price in dollars per cake a bakery can charge if it sells x cakes. What cake price will maximize revenue? (Recall, revenue is the total amount of money received from the sale of x cakes. Make sure to justify why your answer corresponds to an absolute maximum.)

7. A. (7 points) Find the linearization of $g(x) = \ln(x)$ at $x = 1$.

B. (3 points) Use your answer from Part **A** to estimate $\ln(1.15)$.

8. (6 points) The volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$. Find the differential dV.

9. (12 points) A rectangular open-topped aquarium is to have a square base and volume 5 m^3. The material for the base costs \$10 per m^2, and the material for the sides costs \$1 per m^2. What dimensions minimize the cost of the aquarium? (Make sure to justify why your answer corresponds to an absolute minimum.)