EXAM 1: MATH 220 - Calculus 1

June 21st 2017

Name:

Instructor:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
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Instructions: You have 1 hour and 15 minutes to complete this exam. Show all of your work. Calculators are not allowed.

1) (4 points each) Calculate the following limits:

a)
$$\lim_{u \to 4} \frac{\sqrt{u}-2}{4-u} =$$

b)
$$\lim_{x\to 2} (3x^2 + 5x + x^{1/2} - 6) =$$

c)
$$\lim_{\theta \to 0} \frac{\cos(\theta) - \cos^2(\theta)}{4\theta} =$$

d)
$$\lim_{x \to 1} \frac{2x^2 - x - 1}{x - 1} =$$

e)
$$\lim_{x \to 0} \frac{(x+1)\sin(x)}{x} =$$

2) (6 points) Find the limits at infinity for the following functions (Make sure to consider both $+\infty$ and $-\infty$, if necessary.)

a)
$$f(x) = \frac{6x^3 + x^2 - 3}{-2x^3 - 3x^2 + x - 1}$$

b)
$$g(x) = \frac{4x^3 + x + 7}{x - 2}$$

3) (8 points) Show that the equation $2e^t = t^4$ has a solution on the interval [-1, 0]. (Hint: Use the Intermediate Value Theorem)

4) (6 points) Find $\lim_{x\to 0} (5x^2 \cos(\frac{2}{x}))$ using the Squeeze Theorem.

5) (12 points) Let $f(x) = 2x^2 - x$. Compute f'(-1) using the limit definition, and find an equation to the tangent line at a=-1.



6) (16 points) For each point of discontinuity of the graph of the function y=f(x) above, provide the following information:

a) Type of discontinuity

b) The right and left-hand limits.

c) For any removable discontinuity, how would you redefine f(x) so that it is continuous at those points?

d) For any jump disconuity, is f(x) right- or left-continuous at those points?

7) (4 points each) Calculate the following:

a) Find
$$\frac{dy}{dx}$$
 for $y = 7x^4 + 3x^2 - 2x + 1$.

b)
$$f(z) = \frac{3z^2+1}{e^z+2}$$
. Find $f'(z)$.

c)
$$g(x) = (2x^{2/3} - 3x^2)(x^{-1/2} + 5e^x)$$

d)
$$\frac{d}{dx}(\frac{1}{4}x^2 - x^{1/2})$$
 at x=9

e) Let
$$y = x^6 - 3x^4 + x^3 - 3x + 2$$
. Find y".

8) (12 points) Let $s(t) = 50t - \frac{2}{3}t^3$ be a function giving the height of a balloon in feet at time t seconds, $0 \le t \le 8.6$.

a) State/write out the equation for how would you find the velocity function v(t) using the limit definition. Then find v(t) using any method.

b) Find the time t which gives v(t)=0. What is the height of the balloon at this time? (Note: you do not need to simplify your answer).