

## EXAM 2: MATH 220 - Calculus 1

July 11th 2017

Name:

Instructor:

1	2	3	4	5	6	7	8	9	Total

Instructions: You have 1 hour and 15 minutes to complete this exam.  
Show all of your work. Calculators are not allowed.

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1) (16 points) Let  $f(x) = x^3 - 2x^2 + x - 1$ . Find the intervals on which  $f$  is concave up or concave down, increasing or decreasing, the points of inflection, the critical points, and the local minima and maxima.

2) (4 points each) Evaluate the following limits (Hint: you may need to apply L'Hopital's Rule):

a)  $\lim_{\theta \rightarrow \pi} \frac{\cos(\theta)+1}{\theta \sin(\theta)}$

b)  $\lim_{x \rightarrow \infty} (e^{-2x}(x^2 - x - 4))$

3) (8 points) Verify the Mean Value Theorem for the function  $f(x) = x^3 + x$  on the interval  $[-2, 1]$ .

4) (12 points) Find the derivative of  $y = \frac{(x+2)^3}{(x+5)(3x-4)}$  using logarithmic differentiation.

5) (12 points) Compute  $\frac{dy}{dx}$  of  $3xy^2 = y^3 - \cos(x)$  (Hint: use implicit differentiation).

6) (12 points) An architect plans to enclose a 750 square foot rectangular region in a botanical garden. She will use shrubs costing 10 per foot along three sides and fencing costing 5 per foot along the fourth side. Find the minimum total cost.

7) (4 points each) Compute the derivative of the following functions:

a)  $f(x) = \tan(\ln(6x^4 + x^2))$

b)  $y = e^{\sin(x)}$

d)  $g(t) = (5 + (2x^2 - 1)^3)^{1/3}$

8) (12 points) A plane is flying away from you at 500 mph at a height of 3 miles. How fast is the plane's distance from you increasing at the moment when the plane is flying over a point on the ground 4 miles from you?

9) (8 points) Sketch the graph of a function with the following features:  
(i)  $f'(x) > 0$  for  $x < -1$  and  $x > 3$  and  $f'(x) < 0$  for  $-1 < x < 3$ , and  
(ii)  $f''(x) < 0$  for  $x < 1$  and  $f''(x) > 0$  for  $x > 1$