Name Solutions Rec. Instr.

Signature Rec. Time

Math 220
Exam 1
September 20, 2018

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

SHOW YOUR WORK!

Problem	Points	Points Possible	Problem	Points	Points Possible
1		15	7		4
2		5	8		4
3		16	9		7
4		4	10		5
5		7	11		25
6		8	Total Score		100

1. (5 points each) Evaluate the following limits.

A. $\lim\limits_{\theta\to 0} \dfrac{\cos(\theta)}{\theta - 3}$ $= \dfrac{\cos(0)}{0-3} = -\dfrac{1}{3}$

B. $\lim\limits_{x\to 3} \dfrac{x^2 - 5x + 6}{x - 3}$ $= \lim\limits_{x\to 3} \dfrac{(x-2)\cancel{(x-3)}}{\cancel{x-3}} = \lim\limits_{x\to 3} (x-2) = 3-2 = 1$

C. $\lim\limits_{x\to 1} \left(\dfrac{1}{x-1} - \dfrac{1}{x^2 - x} \right)$ $= \lim\limits_{x\to 1} \left(\dfrac{1}{x-1} - \dfrac{1}{x(x-1)} \right) = \lim\limits_{x\to 1} \left(\dfrac{x}{x(x-1)} - \dfrac{1}{x(x-1)} \right)$

$= \lim\limits_{x\to 1} \dfrac{\cancel{x-1}}{x\cancel{(x-1)}} = \lim\limits_{x\to 1} \dfrac{1}{x} = 1$

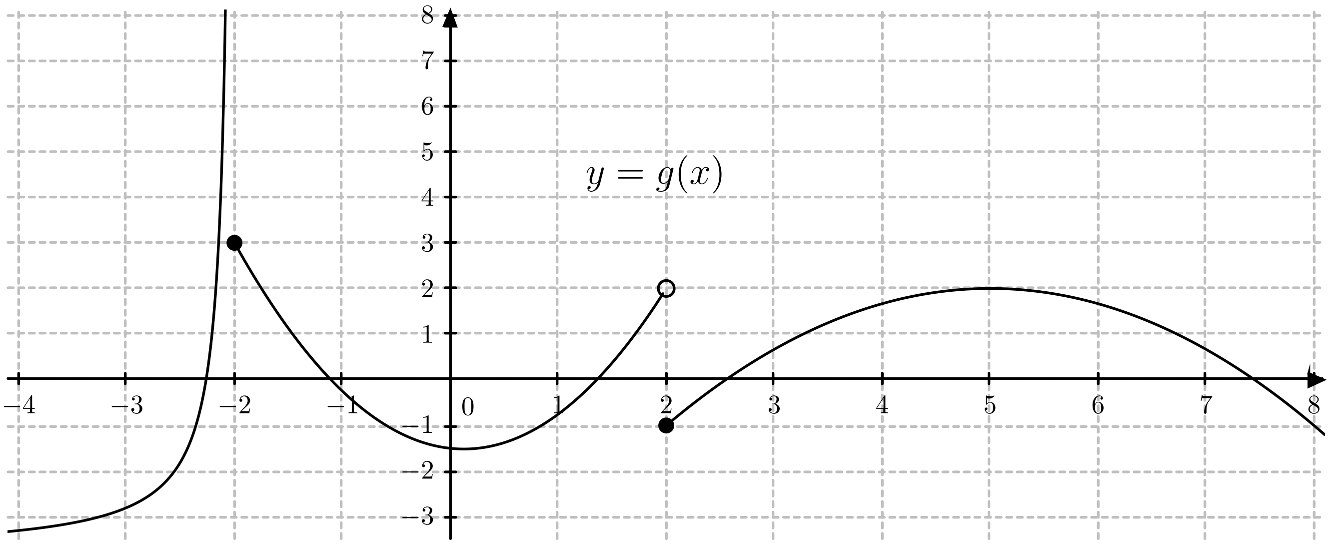
2. (5 points) Evaluate $\lim\limits_{\theta\to 0} \theta^2 \sin\left(\dfrac{1}{\theta}\right)$. **(Justify your reasoning, and state the name of any theorem applied.)**

For $\theta \neq 0$, $-1 \leq \sin\left(\frac{1}{\theta}\right) \leq 1$ and $\theta^2 > 0$,

implying that $-\theta^2 \leq \theta^2 \sin\left(\frac{1}{\theta}\right) \leq \theta^2$.

$\lim\limits_{\theta\to 0} -\theta^2 = -0^2 = 0$. $\lim\limits_{\theta\to 0} \theta^2 = 0^2 = 0$.

By the Squeeze Theorem, $\lim\limits_{\theta\to 0} \theta^2 \sin\left(\frac{1}{\theta}\right) = 0$.



3. (2 points each) Consider the graph of $y = g(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x \to -2^-} g(x) = \infty$

B. $\lim_{x \to -2^+} g(x) = 3$

C. $\lim_{x \to -2} g(x)$ does not exist

D. $g(-2) = 3$

E. $\lim_{x \to 2^-} g(x) = 2$

F. $\lim_{x \to 2^+} g(x) = -1$

G. $\lim_{x \to 2} g(x)$ does not exist

H. $g(2) = -1$

4. (4 points) Consider the graph of $y = g(x)$ above. List the x-coordinates where the function is discontinuous.

$x = -2$ and $x = 2$

5. (7 points) Let $f(x) = \sqrt{x}$. **Using the limit definition of the derivative**, find $f'(4)$.

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h \to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h} = \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{-\sqrt{4+h} - 2}{-\sqrt{4+h} - 2}$$

$$= \lim_{h \to 0} \frac{-(\cancel{4}+h) \cancel{-2\sqrt{4+h}} \cancel{+2\sqrt{4+h}} \cancel{+4}}{h(-\sqrt{4+h} - 2)}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(-\sqrt{4+h} - 2)} = \lim_{h \to 0} \frac{-1}{-\sqrt{4+h} - 2}$$

$$= \frac{-1}{-\sqrt{4+0} - 2} = \frac{1}{4}$$

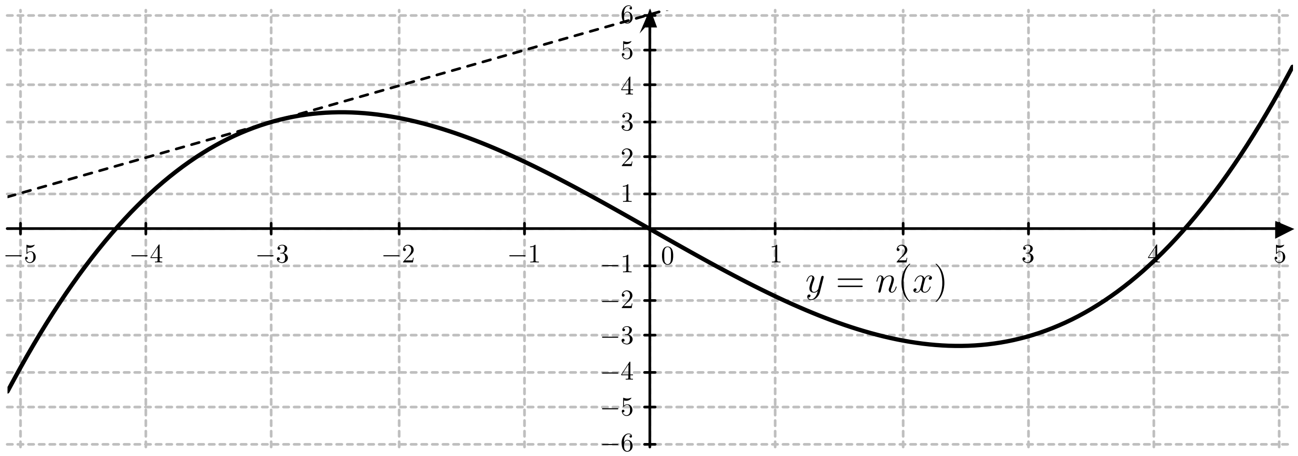
6. (4 points each) Given that $\lim_{x \to -2} u(x) = 5$ and $\lim_{x \to -2} w(x) = 11$, find the following limits.

A. $\lim_{x \to -2} \frac{w(x) + 4}{u(x)}$ $= \frac{11+4}{5} = \frac{15}{5} = 3$

B. $\lim_{x \to -2} \sqrt{u(x) + w(x)}$ $= \sqrt{5+11} = \sqrt{16} = 4$

7. (4 points) Suppose that a waiter brings you a cold beverage on a hot day. Let $T(t)$ denote the temperature in degrees Fahrenheit of the beverage after t minutes. Is $T'(2)$ positive or negative? (Explain your answer.)

$T'(2)$ is positive because the temperature of the beverage is increasing two minutes after you get it.



8. (2 points each) The function $y = n(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values.

A. $n(-3) =$ 3 **B.** $n'(-3) =$ 1

9. The height in feet of a ball t seconds after being thrown directly upward is given by $y(t) = -16t^2 + 20t + 5$.

A. (5 points) Find the velocity 1 second after the ball is thrown (**include the units**).

$y'(t) = -32t + 20$

$y'(1) = -32 \cdot 1 + 20 = -12$ ft/s

B. (2 points) Is the ball going upward or downward 1 second after being thrown?

It is going downward because the velocity is negative.

10. (5 points) Given that $h(2) = 5$ and $h'(2) = 3$, find the equation of the tangent line to the graph of $y = h(x)$ at $x = 2$.

This is a line of slope $h'(2) = 3$ that goes through the point $(2, h(2)) = (2, 5)$.

$y - 5 = 3(x - 2)$

(In slope-intercept form, it is $y = 3x - 1$.)

11. (5 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\frac{d}{dx}\left(3x^8 - \pi^2\right) = 3 \cdot 8x^7 - 0 = 24x^7$

B. $\frac{d}{dx}\left(\frac{5}{x^3} + \sqrt{x}\right) = \frac{d}{dx}\left(5x^{-3} + x^{1/2}\right) = 5\cdot(-3)x^{-4} + \frac{1}{2}x^{-1/2}$

$= \frac{-15}{x^4} + \frac{1}{2\sqrt{x}}$

C. $\frac{d}{d\theta}\left(\sin(\theta)\cdot\cos(\theta)\right) = \left[\frac{d}{d\theta}\sin(\theta)\right]\cdot\cos(\theta) + \sin(\theta)\left[\frac{d}{d\theta}\cos(\theta)\right]$

$= \cos(\theta)\cdot\cos(\theta) + \sin(\theta)\cdot(-\sin(\theta))$

$= \cos^2(\theta) - \sin^2(\theta)$

D. $\frac{d}{d\theta}\tan\left(2\theta^7\right) = \sec^2(2\theta^7)\cdot\left[\frac{d}{d\theta}2\theta^7\right] = \sec^2(2\theta^7)\cdot 2\cdot 7\cdot\theta^6$

$= 14\cdot\sec^2(2\theta^7)\cdot\theta^6$

E. $\frac{d}{dx}\left(\frac{x^5+4}{3x^2+9}\right) = \frac{\left[\frac{d}{dx}(x^5+4)\right]\cdot(3x^2+9) - (x^5+4)\cdot\left[\frac{d}{dx}(3x^2+9)\right]}{(3x^2+9)^2}$

$= \frac{5x^4\cdot(3x^2+9) - (x^5+4)\cdot 6x}{(3x^2+9)^2}$