Name Solutions Rec. Instr.
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Math 220
Exam 2
October 18, 2018

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		8	7		3
2		10	8		12
3		10	9		6
4		10	10		10
5		10	11		15
6		6	**Total Score**		100

1. (4 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\frac{d}{dx}\left(x \cdot \arctan(3x^2)\right) = \frac{d}{dx}\left(x \cdot \tan^{-1}(3x^2)\right)$

$$= 1 \cdot \arctan(3x^2) + x \cdot \frac{1}{1+(3x^2)^2} \cdot 6x$$

$$= \arctan(3x^2) + \frac{6x^2}{1+9x^4}$$

B. $\frac{d}{dx}\left(\frac{2^x - \ln(x)}{e^x + 1}\right) = \frac{\left(2^x \cdot \ln(2) - \frac{1}{x}\right)\cdot(e^x+1) - (2^x - \ln(x))\cdot e^x}{(e^x+1)^2}$

2. A. (7 points) Find the linearization of $g(x) = \sqrt{x}$ at $x = 25$.

$$g'(x) = \frac{1}{2\sqrt{x}}$$

$$L(x) = g(25) + g'(25)(x-25)$$

$$= \sqrt{25} + \frac{1}{2\sqrt{25}}(x-25)$$

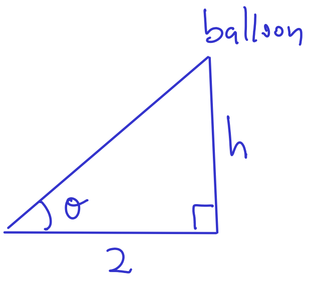
$$= 5 + \frac{1}{10}(x-25)$$

B. (3 points) Use your answer from Part **A** to estimate $\sqrt{26}$.

$$\sqrt{26} = g(26) \underset{\uparrow}{\approx} L(26) = 5 + \frac{1}{10}(26-25) = 5.1$$

26 is close to 25

3. (10 points) A hot air balloon rising vertically is tracked by an observer located 2 miles from the lift-off point. At a certain moment, the angle between the observer's line of sight and the horizontal is $\frac{\pi}{4}$, and it is changing at a rate of $\frac{1}{10}$ radians/minute. How fast is the balloon rising at this moment? (Include the units.)



Want: $\frac{dh}{dt}$ when $\theta=\frac{\pi}{4}$ and $\frac{d\theta}{dt}=\frac{1}{10}$ rad/min

$$\tan(\theta)=\frac{h}{2}$$

$$h=2\tan(\theta)$$

$$\frac{dh}{dt}=2\sec^2(\theta)\frac{d\theta}{dt}$$

When $\theta=\frac{\pi}{4}$ and $\frac{d\theta}{dt}=\frac{1}{10}$ rad/min, we have

$$\frac{dh}{dt}=2\sec^2\left(\frac{\pi}{4}\right)\cdot\frac{1}{10}=\frac{1}{5\cos^2(\frac{\pi}{4})}=\frac{1}{5\cdot(\frac{1}{\sqrt{2}})^2}=\frac{2}{5}\text{ mi/min}$$

4. (10 points) If $x^2y-e^y=x+1$, compute $\frac{dy}{dx}$ in terms of x and y.

$$\frac{d}{dx}(x^2y-e^y)=\frac{d}{dx}(x+1)$$

$$2x\cdot y+x^2\cdot\frac{dy}{dx}-e^y\cdot\frac{dy}{dx}=1$$

$$x^2\cdot\frac{dy}{dx}-e^y\frac{dy}{dx}=1-2xy$$

$$(x^2-e^y)\frac{dy}{dx}=1-2xy$$

$$\frac{dy}{dx}=\frac{1-2xy}{x^2-e^y}$$

5. (10 points) Find the absolute maximum and absolute minimum of $w(x) = x + \sin(x)$ on $[0, 2\pi]$.

$w'(x) = 1 + \cos(x)$ is defined everywhere.

$w'(x) = 0 \iff \cos(x) = -1 \iff x = \pi + 2\pi k$

The only critical number in $(0, 2\pi)$ is $x = \pi$

$w(0) = 0 + \sin(0) = 0$

$w(\pi) = \pi + \sin(\pi) = \pi$

$W(2\pi) = 2\pi + \sin(2\pi) = 2\pi$

On $[0, 2\pi]$, the absolute max of $w(x)$ is $w(2\pi) = 2\pi$ and the absolute minimum of $w(x)$ is $w(0) = 0$.

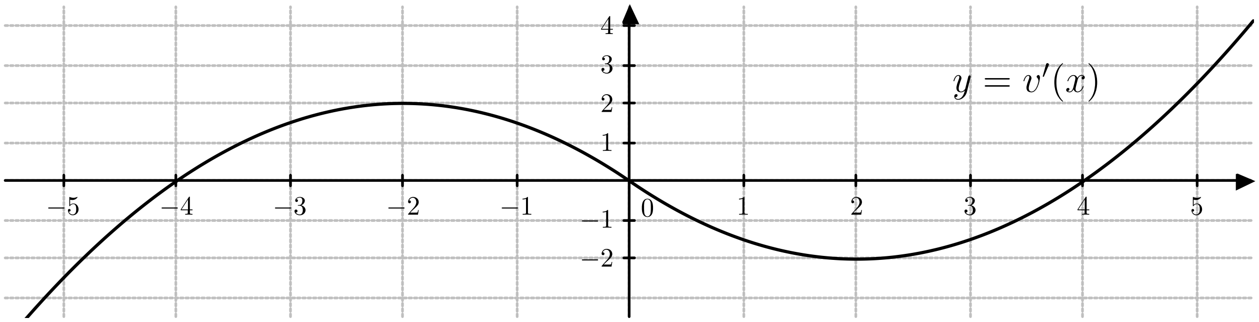
6. (6 points) Let V denote the volume of a cube of side length x. Find the differential dV in terms of x and dx.

$V = x^3$

$\frac{dV}{dx} = 3x^2$

$dV = 3x^2 \cdot dx$

7. (3 points) Find $\lim_{x \to -\infty} \frac{3x^9 - 7x + 3}{2 + 5x + 6x^9}$. $= \frac{3}{6}\left(\lim_{x \to -\infty} x^{9-9}\right) = \frac{3}{6} = \frac{1}{2}$



8. (3 points each) $y = v'(x)$ is plotted above. Find:

A. Interval(s) where $v(x)$ is increasing: $(-4,0), (4,\infty)$ decreasing: $(-\infty,-4), (0,4)$

B. x-coordinate(s) where $v(x)$ has a local max: 0 local min: $-4, 4$

C. Interval(s) where $v(x)$ is concave up: $(-\infty,-2), (2,\infty)$ concave down: $(-2, 2)$

D. x-coordinate(s) where $v(x)$ has an inflection point: $-2, 2$

9. (3 points each) In each of the following blanks, fill in "**max**" or "**min**".

A. If $h'(7) = 0$ and $h''(7) = 4$, then $h(x)$ has a local min at $x = 7$.

B. If $h'(5) = 0$ and $h''(5) = -.33$, then $h(x)$ has a local max at $x = 5$.

10. (10 points) Find the derivative of $h(x) = x^{5x^2}$.

$$\ln(h(x)) = \ln(x^{5x^2}) = 5x^2 \ln(x)$$

$$\frac{d}{dx} \ln(h(x)) = \frac{d}{dx} 5x^2 \cdot \ln(x)$$

$$\frac{h'(x)}{h(x)} = 10x \cdot \ln(x) + 5x^2 \cdot \frac{1}{x} = 10x \ln(x) + 5x$$

$$h'(x) = h(x) \cdot (10x \ln(x) + 5x)$$

$$h'(x) = x^{5x^2} \cdot (10x \ln(x) + 5x)$$

11. The function $f(x)$ and its first and second derivatives are:

$$f(x) = x^2(x+3) \qquad f'(x) = 3x(x+2) \qquad f''(x) = 6(x+1).$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: $(-\infty, \infty)$

B. (1 point) y-intercept: 0

C. (1 point) x-intercept(s): $-3, 0$

D. (1 point) Interval(s) $f(x)$ is increasing: $(-\infty, -2), (0, \infty)$

E. (1 point) Interval(s) $f(x)$ is decreasing: $(-2, 0)$

$f'(x)$ is always defined. $f'(x) = 0$ when $x = -2$ or $x = 0$.

Sign of $f'(x)$ + − +

-2 0

F. (1 point) Location(s) where $f(x)$ has a local max: $x = -2$, $f(-2) = 4$

G. (1 point) Location(s) where $f(x)$ has a local min: $x = 0$, $f(0) = 0$

H. (1 point) Interval(s) $f(x)$ is concave up: $(-1, \infty)$

I. (1 point) Interval(s) $f(x)$ is concave down: $(-\infty, -1)$

J. (1 point) Inflection point(s) (x, y): $(-1, 2)$

K. (5 points) Sketch $y = f(x)$ on the graph below.