Name Solutions Rec. Instr.

Signature Rec. Time

Math 220

Exam 3

November 15, 2018

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		25	5		10
2		12	6		8
3		10	7		14
4		10	8		11

Total Score	

1. (5 points each) Evaluate the following:

A. $\lim_{\theta \to 0} \frac{\sin(\theta^2)}{\theta^2} = \underset{\left(\substack{LH \\ \frac{0}{0}}\right)}{} \lim_{\theta \to 0} \frac{\cos(\theta^2)\cdot \cancel{2\theta}}{\cancel{2\theta}} = \lim_{\theta \to 0} \cos(\theta^2) = \cos(0^2) = 1$

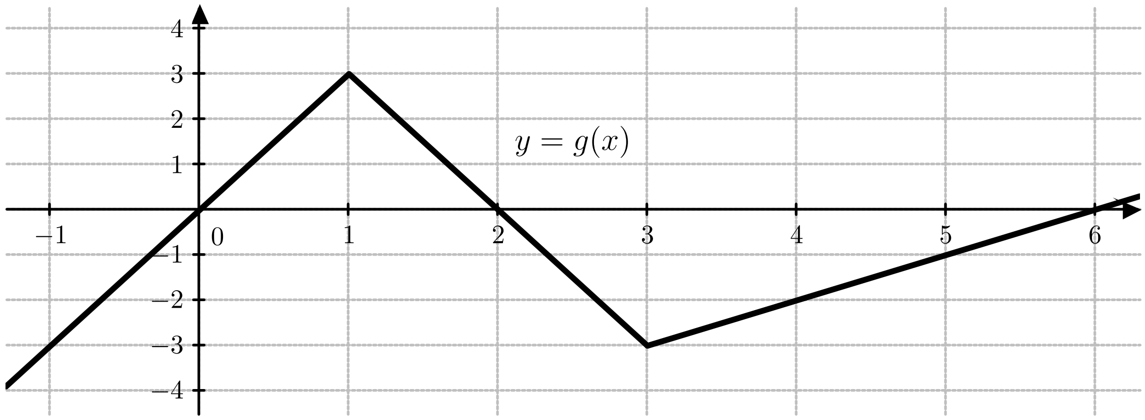
B. $\lim_{x \to \infty} \frac{x \ln(x)}{x^2 + 3x} = \underset{\left(\substack{LH \\ \frac{\infty}{\infty}}\right)}{} \lim_{x \to \infty} \frac{1 \cdot \ln(x) + x \cdot \frac{1}{x}}{2x+3} = \lim_{x \to \infty} \frac{\ln(x) + 1}{2x+3} \underset{\left(\substack{LH \\ \frac{\infty}{\infty}}\right)}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{2} = \frac{0}{2} = 0$

C. $\int \left(\sqrt{x} + 6\sec^2(x) - 5\right)\, dx = \frac{2}{3} x^{3/2} + 6\tan(x) - 5x + C$

D. $\frac{d}{dx} \int_3^x e^{2t} \sin(t^3)\, dt = e^{2x} \sin(x^3)$

E. $\int_0^4 (e^x - 3)\, dx = (e^x - 3x)\Big|_0^4 = (e^4 - 3\cdot 4) - (e^0 - 3\cdot 0)$

$= (e^4 - 12) - (1 - 0) = e^4 - 13$



2. (4 points each) $y = g(x)$ is plotted above. Let $A(x) = \int_0^x g(t)\,dt$. Find the following quantities.

A. $\int_2^6 g(x)\,dx = -\frac{1}{2}\cdot 4\cdot 3 = -6$

B. $\int_1^0 g(x)\,dx = -\int_0^1 g(x)\,dx = -\frac{1}{2}\cdot 1\cdot 3 = -\frac{3}{2}$

C. $A'(4) = g(4) = -2$ because $A'(x) = \frac{d}{dx}\int_0^x g(t)\,dt = g(x)$

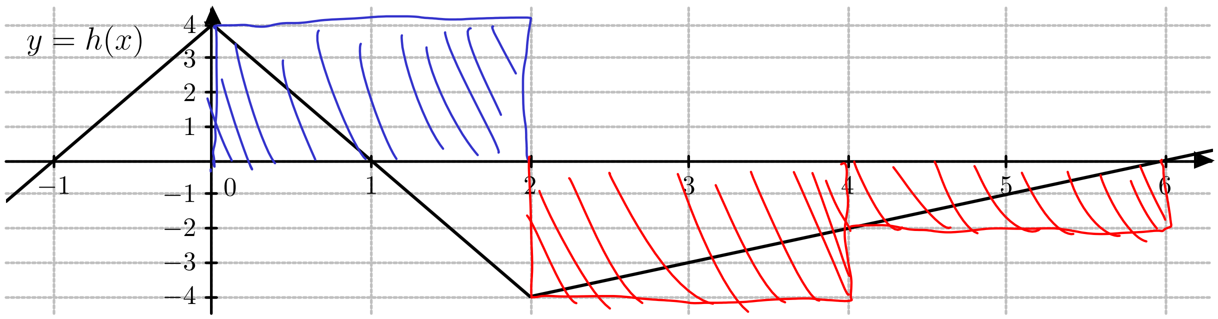
3. (10 points) Find $f(x)$ if $f''(x) = 6x$, $f'(0) = 1$, and $f(0) = 2$.

$f'(x) = 3x^2 + C$ for some constant C.

$1 = f'(0) = 3\cdot 0^2 + C = C$ so $f'(x) = 3x^2 + 1$

$f(x) = x^3 + x + D$ for some constant D.

$2 = f(0) = 0^3 + 0 + D = D$ so $f(x) = x^3 + x + 2$



4. (10 points) Estimate $\int_0^6 h(x)\,dx$ by computing L_3, the Left-Endpoint Approximation with 3 subintervals. Also, illustrate the rectangles on the graph above.

$$\Delta x = \frac{6-0}{3} = 2$$

$$\int_0^6 h(x)\,dx \approx h(0)\Delta x + h(2)\Delta x + h(4)\Delta x$$

$$= 4\cdot 2 - 4\cdot 2 - 2\cdot 2 = -4$$

5. Let $w(t) = 4 - t/2$ for $0 \le t \le 8$ be the rate that water flows out of a storage tank in gallons per minute at time t minutes after the tank ruptures.

A. (7 points) Find $\int_2^4 w(t)\,dt$. (Include units with your answer.)

$$\int_2^4 w(t)\,dt = \int_2^4 \left(4 - \frac{t}{2}\right)dt = \left(4t - \frac{t^2}{4}\right)\Big|_2^4$$

$$= \left(4\cdot 4 - \frac{4^2}{4}\right) - \left(4\cdot 2 - \frac{2^2}{4}\right)$$

$$= (16-4) - (8-1)$$

$$= 5 \text{ gallon}$$

B. (3 points) What does $\int_2^4 w(t)\,dt$ represent?

By the Net Change Theorem, this represents the total amount of water that flows out of the tank from 2 minutes after the rupture to 4 minutes after the rupture

6. (8 points) Suppose that a particle has position $s(t)$ feet at time t seconds and a velocity function $s'(t) = 3\cos(t)$ ft/s. Find the displacement (change in position) from time $t = 0$ seconds to time $t = \pi/2$ seconds. (Include units with your answer.)

$$s(\tfrac{\pi}{2}) - s(0) = \int_0^{\pi/2} 3\cos(t)\,dt = 3\sin(t)\Big|_0^{\pi/2}$$
$$= 3\sin(\tfrac{\pi}{2}) - 3\sin(0)$$
$$= 3 \text{ ft.}$$

7. (7 points each) Evaluate the following:

A. $\displaystyle\int_0^{\pi/2} 2\sin^3(\theta)\cos(\theta)\,d\theta = \int_0^1 2u^3\,du = 2\cdot\frac{1}{4}u^4\Big|_0^1 = \frac{u^4}{2}\Big|_0^1$

$$= \frac{1^4}{2} - \frac{0^4}{2} = \frac{1}{2}$$

$u = \sin(\theta)$

$\frac{du}{d\theta} = \cos(\theta)$

$du = \cos(\theta)\,d\theta$

θ	u
$\frac{\pi}{2}$	1
0	0

B. $\displaystyle\int x\sqrt{5+x}\,dx = \int (u-5)\sqrt{u}\,du = \int \left(u^{3/2} - 5u^{1/2}\right)du$

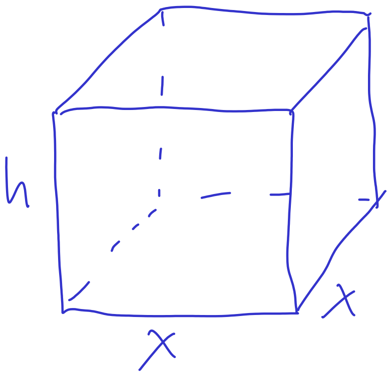
$$= \frac{2}{5}u^{5/2} - 5\cdot\frac{2}{3}u^{3/2} + C$$
$$= \frac{2}{5}u^{5/2} - \frac{10}{3}u^{3/2} + C$$
$$= \frac{2}{5}(5+x)^{5/2} - \frac{10}{3}(5+x)^{3/2} + C$$

$u = 5 + x$

$du = dx$

$x = u - 5$

8. (11 points) A rectangular open-topped aquarium is to have a square base and volume 8 m^3. The material for the base costs \$2 per m^2, and the material for the sides costs \$1 per m^2. What dimensions minimize the cost of the aquarium? (Make sure to justify why your answer corresponds to an absolute minimum. Include units with your answer.)



We want to minimize the cost

$C = 1 \cdot 4 \cdot xh + 2 \cdot x^2 = 4xh + 2x^2$

subject to the constraint Volume $= x^2 h = 8$.

$h = \frac{8}{x^2}$ so $C(x) = 4x \cdot \frac{8}{x^2} + 2x^2 = \frac{32}{x} + 2x^2$

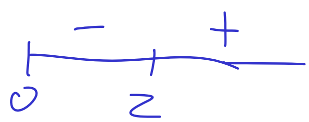
We want to minimize $C(x)$ on $(0,\infty)$.

$C'(x) = -\frac{32}{x^2} + 4x$ is defined on $(0,\infty)$.

$C'(x) = 0$ when $4x = \frac{32}{x^2} \Rightarrow 4x^3 = 32 \Rightarrow x^3 = 8 \Rightarrow x = 2$

Justification with 1st Derivative

Sign of $C'(x)$



$C(x)$ is decreasing on $(0,2)$ and increasing on $(2,\infty)$.

Justication with 2nd Derivative

$C''(x) = \frac{64}{x^3} > 0$ on $(0,\infty)$

$C''(x)$ is concave up on $(0,\infty)$

Cost is minimized when $x = 2$ m and $h = \frac{8}{2^2} = 2$ m.

The dimensions are 2m by 2m by 2m.