Name Solutions Rec. Instr.
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Math 220
Exam 1
September 26, 2019

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

SHOW YOUR WORK!

Problem	Points	Points Possible	Problem	Points	Points Possible
1		15	7		4
2		5	8		4
3		16	9		7
4		4	10		5
5		7	11		25
6		8	Total Score		100

1. (5 points each) Evaluate the following limits.

A. $\lim_{\theta \to \pi} \frac{1 - \sin(\theta)}{\theta}$ $= \frac{1-\sin(\pi)}{\pi} = \frac{1}{\pi}$

B. $\lim_{x \to -2} \frac{x^2 - 4}{x + 2}$ $= \lim_{x\to -2} \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}} = \lim_{x\to -2} (x-2)$

$= -2-2 = -4$

C. $\lim_{t \to 1} \frac{\sqrt{t} - 1}{t - 1}$ $= \lim_{t\to 1} \frac{\sqrt{t}-1}{t-1} \cdot \frac{\sqrt{t}+1}{\sqrt{t}+1} = \lim_{t\to 1} \frac{t + \cancel{\sqrt{t}} - \cancel{\sqrt{t}} - 1}{(t-1)(\sqrt{t}+1)}$

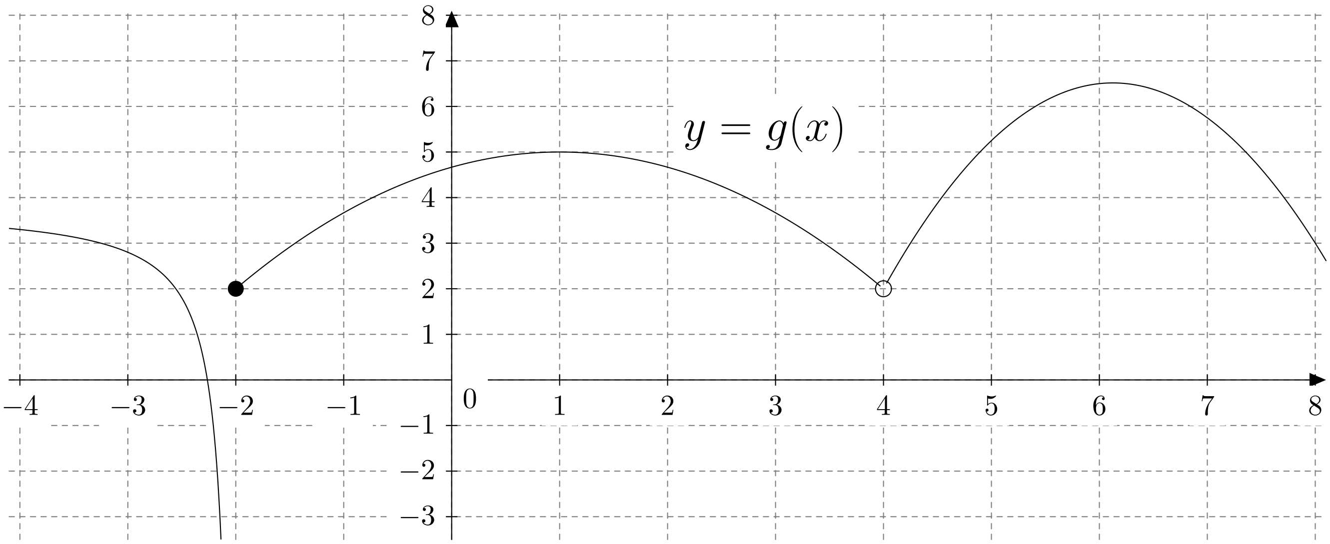
$= \lim_{t\to 1} \frac{\cancel{t-1}}{\cancel{(t-1)}(\sqrt{t}+1)} = \lim_{t\to 1} \frac{1}{(\sqrt{t}+1)} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

2. (5 points) Provided that $10x - 25 \leq h(x) \leq x^2$ for all $x \neq 5$, find $\lim_{x \to 5} h(x)$. **(Justify your reasoning, and state the name of any theorem applied.)**

$\lim_{x\to 5} (10x-25) = 10\cdot 5 - 25 = 25$

$\lim_{x\to 5} x^2 = 5^2 = 25$

By the Squeeze Theorem, $\lim_{x\to 5} h(x) = 25.$



3. (2 points each) Consider the graph of $y = g(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x \to -2^-} g(x) = -\infty$

B. $\lim_{x \to -2^+} g(x) = 2$

C. $\lim_{x \to -2} g(x)$ does not exist

D. $g(-2) = 2$

E. $\lim_{x \to 4^-} g(x) = 2$

F. $\lim_{x \to 4^+} g(x) = 2$

G. $\lim_{x \to 4} g(x) = 2$

H. $g(4)$ does not exist

4. (4 points) Consider the graph of $y = g(x)$ above. List the x-coordinates where the function is discontinuous.

$x = -2$ and $x = 4$

5. (7 points) Let $f(x) = \frac{1}{x}$. **Using the limit definition of the derivative**, find $f'(3)$.

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

$$= \lim_{h\to 0} \left(\frac{1}{(3+h)h} - \frac{1}{3h} \right) = \lim_{h\to 0} \left(\frac{3}{3(3+h)h} - \frac{3+h}{3(3+h)h} \right)$$

$$= \lim_{h\to 0} \frac{\cancel{3} - \cancel{3} - h}{3(3+h)h} = \lim_{h\to 0} \frac{-\cancel{h}}{3(3+h)\cancel{h}} = \lim_{h\to 0} \frac{-1}{3(3+h)}$$

$$= \frac{-1}{3(3+0)} = -\frac{1}{9}$$

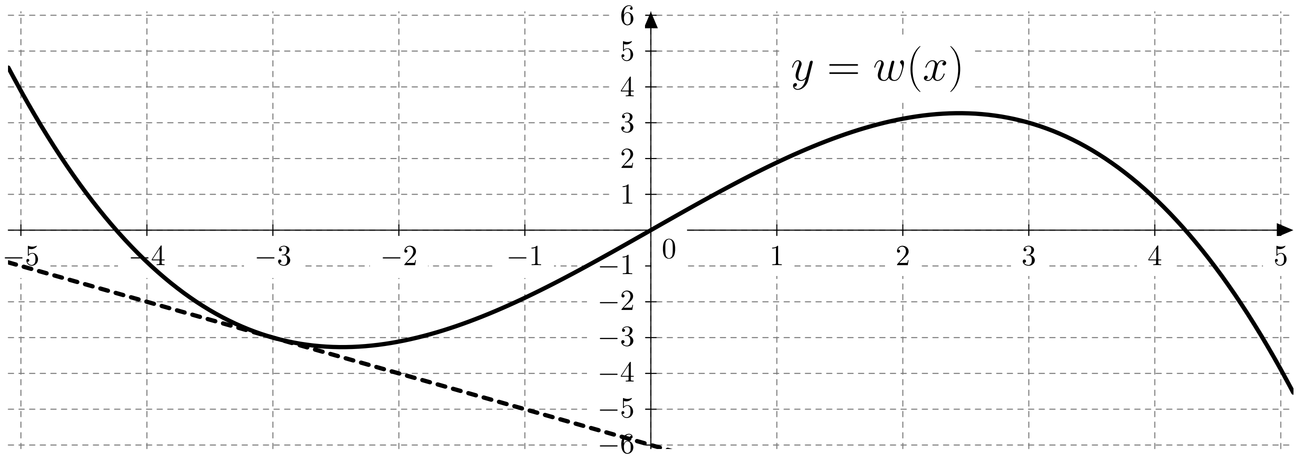
6. (4 points each) Given that $\lim_{x\to 6} q(x) = 2$ and $\lim_{x\to 6} r(x) = \pi$, find the following limits.

A. $\lim_{x\to 6} \frac{r(x)}{x + q(x)}$ $= \frac{\pi}{6+2} = \frac{\pi}{8}$

B. $\lim_{x\to 6} \cos\big(q(x) \cdot r(x)\big)$ $= \cos(2\pi) = 1$

7. (4 points) The height of a plane in meters at time t seconds after the plane elevates off of the ground at takeoff is given by $h(t)$. Is $h'(1)$ positive or negative? Briefly explain your answer.

$h'(1)$ is positive because the plane is elevating at time $t=1$ second.



8. (2 points each) The function $y = w(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values.

A. $w(-3) =$ -3 **B.** $w'(-3) =$ -1

9. The height in feet of a ball t seconds after being thrown directly upward is given by $y(t) = -16t^2 + 40t + 5$.

A. (5 points) Find the velocity 1 second after the ball is thrown (**include the units**).

$y'(t) = -32t + 40$

$y'(1) = -32 \cdot 1 + 40 = 8$ ft/s

B. (2 points) Is the ball going upward or downward 1 second after being thrown?

upward because $y'(1)$ is positive.

10. (5 points) Given that $v(5) = 3$ and $v'(5) = -2$, find an equation of the tangent line to the graph of $y = v(x)$ at $x = 5$.

The line has slope $v'(5) = -2$ and goes through $(5, 3)$.

$y - 3 = -2(x - 5)$

11. (5 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\dfrac{d}{dx}\left(5x^2 + 3x^{1/3} + 3\pi\right) = 5\cdot 2x + 3\cdot\frac{1}{3}x^{-2/3} + 0$

$= 10x + x^{-2/3}$

B. $\dfrac{d}{dx}\left(\dfrac{7}{\sqrt{x}} + 2\sqrt{x}\right) = \frac{d}{dx}\left(7x^{-1/2} + 2x^{1/2}\right) = 7\cdot\left(\frac{-1}{2}\right)x^{-3/2} + 2\cdot\frac{1}{2}x^{-1/2}$

$= -\frac{7}{2}x^{-3/2} + x^{-1/2}$

C. $\dfrac{d}{d\theta}\left(\theta\cdot\tan(\theta^3)\right) = \left[\frac{d}{d\theta}\theta\right]\tan(\theta^3) + \theta\cdot\left[\frac{d}{d\theta}\tan(\theta^3)\right]$

$= 1\cdot\tan(\theta^3) + \theta\cdot\sec^2(\theta^3)\cdot\left[\frac{d}{d\theta}\theta^3\right]$

$= \tan(\theta^3) + \theta\sec^2(\theta^3)\cdot 3\theta^2$

$= \tan(\theta^3) + 3\theta^3\sec^2(\theta^3)$

D. $\dfrac{d}{d\theta}\cos\left(\sin(\theta^2)\right) = -\sin\left(\sin(\theta^2)\right)\left[\frac{d}{d\theta}\sin(\theta^2)\right]$

$= -\sin(\sin(\theta^2))\cdot\cos(\theta^2)\cdot\left[\frac{d}{d\theta}\theta^2\right]$

$= -\sin(\sin(\theta^2))\cdot\cos(\theta^2)\cdot 2\theta$

$= -2\theta\sin(\sin(\theta^2))\cdot\cos(\theta^2)$

E. $\dfrac{d}{dx}\left(\dfrac{x^{3/2} - 5}{x^2 + 1}\right) = \dfrac{\left[\frac{d}{dx}(x^{3/2} - 5)\right]\cdot(x^2+1) - (x^{3/2} - 5)\cdot\left[\frac{d}{dx}(x^2+1)\right]}{(x^2+1)^2}$

$= \dfrac{\left(\frac{3}{2}\cdot\sqrt{x}\right)\cdot(x^2+1) - (x^{3/2} - 5)\cdot(2x)}{(x^2+1)^2}$