Name Solutions Rec. Instr.

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Math 220

Exam 2

October 24, 2019

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		12	6		10
2		8	7		12
3		10	8		6
4		10	9		6
5		10	10		16

Total Score	

1. (4 points each) Find the following quantities. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect. Recall that $\arctan(z)$ is the inverse trigonometric function $\tan^{-1}(z)$.

A. $\dfrac{d}{dx}\left(\ln(x^3+7)\cdot \arctan(x^2+5)\right)$

$$= \frac{3x^2}{x^3+7}\cdot \arctan(x^2+5) + \ln(x^3+7)\cdot \frac{1}{1+(x^2+5)^2}\cdot 2x$$

B. $\dfrac{d}{dx}\left(\dfrac{2^x+5}{e^x+1}\right) = \dfrac{2^x\cdot\ln(2)\cdot(e^x+1)-(2^x+5)\cdot e^x}{(e^x+1)^2}$

C. $\displaystyle\lim_{x\to-\infty}\frac{2x^8-x^2+3}{7x^9+14x+6} = \frac{2}{7}\left[\lim_{x\to-\infty} x^{8-9}\right] = \frac{2}{7}\cdot 0 = 0$

2. (8 points) Let $s(t)$ denote the position of a particle in cm after t seconds. Suppose that we are unable to write down a nice formula for $s(t)$, but we happen to know that when $t = 1$ second, the particle has position 5 cm and velocity 2 cm/s. Find the linearization of $s(t)$ at $t = 1$, and use it to approximate the location of the particle when $t = .9$ seconds. (Include units with your answer.)

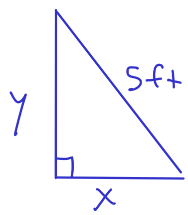
The linearization of $s(t)$ at $t=1$ is

$L(t) = s(1) + s'(1)(t-1) = 5 + 2(t-1)$.

$s(.9) \approx L(.9) = 5 + 2(.9-1) = 5 - .2 = 4.8$ cm

.9 is close to 1

3. (10 points) A 5 foot ladder is leaning against a wall. If the top slips down the wall at a rate of 2 ft/s, how fast will the bottom of the ladder be moving away from the wall when the top is 3 feet above the ground? (Include the units.)



Want: $\frac{dx}{dt}$ when $y = 3$ ft

Know: $\frac{dy}{dt} = -2$ ft/s

$$x^2 + y^2 = 5^2$$

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt} 5^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2x \frac{dx}{dt} = -2y \frac{dy}{dt}$$

$$\frac{dx}{dt} = -\frac{2y}{2x} \frac{dy}{dt} = -\frac{y}{x} \frac{dy}{dt}$$

When $y = 3$ft, $x = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$ft and $\frac{dx}{dt} = -\frac{3}{4} \cdot (-2) = \frac{3}{2}$ ft/s.

4. (10 points) If $\sin(xy) = x^2$, compute $\dfrac{dy}{dx}$ in terms of x and y.

$$\frac{d}{dx} \sin(xy) = \frac{d}{dx} x^2$$

$$\cos(xy) \left[\frac{d}{dx} xy\right] = 2x$$

$$\cos(xy)\left[1 \cdot y + x \cdot \frac{dy}{dx}\right] = 2x$$

$$y\cos(xy) + x\cos(xy)\frac{dy}{dx} = 2x$$

$$x\cos(xy)\frac{dy}{dx} = 2x - y\cos(xy)$$

$$\frac{dy}{dx} = \frac{2x - y\cos(xy)}{x\cos(xy)}$$

5. (10 points) Find the absolute maximum and absolute minimum of $w(x) = x^3 - 3x + 2$ on $[0, 2]$.

$w'(x) = 3x^2 - 3 = 3(x^2-1) = 3(x-1)(x+1)$ is always defined on $[0,2]$

$0 = w'(x)$ implies $x = \pm 1$, but only $x=1$ is in $(0,2)$.

$w(0) = 0^3 - 3 \cdot 0 + 2 = 2$

$w(1) = 1^3 - 3 \cdot 1 + 2 = 0$

$w(2) = 2^3 - 3 \cdot 2 + 2 = 4$

$w(x)$ has an absolute max $w(2) = 4$

and an absolute min $w(1) = 0$.

6. (10 points) Find the derivative of $h(x) = x^{\cos(x)}$.

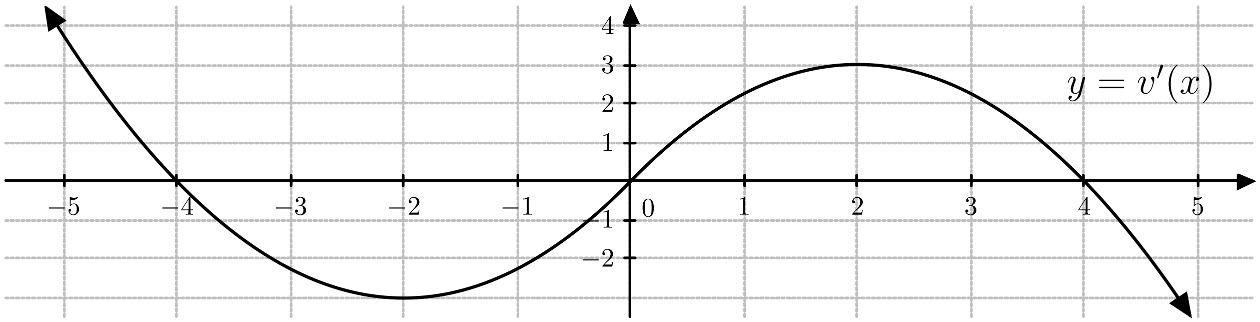
$$\ln(h(x)) = \ln(x^{\cos(x)}) = \cos(x) \cdot \ln(x)$$

$$\frac{d}{dx} \ln(h(x)) = \frac{d}{dx} \cos(x) \ln(x)$$

$$\frac{h'(x)}{h(x)} = -\sin(x) \cdot \ln(x) + \cos(x) \cdot \frac{1}{x}$$

$$h'(x) = h(x) \left(-\sin(x) \cdot \ln(x) + \frac{\cos(x)}{x} \right)$$

$$h'(x) = x^{\cos(x)} \left(-\sin(x) \ln(x) + \frac{\cos(x)}{x} \right)$$



7. (3 points each) $y = v'(x)$ is plotted above. Find:

A. Interval(s) where $v(x)$ is increasing: $(-\infty,-4),(0,4)$ decreasing: $(-4,0),(4,\infty)$

B. x-coordinate(s) where $v(x)$ has a local max: $-4, 4$ local min: 0

C. Interval(s) where $v(x)$ is concave up: $(-2,2)$ concave down: $(-\infty,-2),(2,\infty)$

D. x-coordinate(s) where $v(x)$ has an inflection point: $-2, 2$

8. (3 points each) In each of the following blanks, fill in "**max**" or "**min**".

A. If $h'(2) = 0$ and $h''(2) = -\dfrac{\pi}{3}$, then $h(x)$ has a local max at $x = 2$.

B. If $h'(7) = 0$ and $h''(7) = \sqrt{2}$, then $h(x)$ has a local min at $x = 7$.

9. (6 points) The volume of a sphere is $V = \dfrac{4}{3}\pi r^3$, where r denotes the radius. Find the differential dV in terms of r and dr.

$$\frac{dV}{dr} = \frac{4}{3}\cdot\pi\cdot 3r^2 = 4\pi r^2$$

$$dV = 4\pi r^2 dr$$

10. The function $f(x)$ and its first and second derivatives are:

$$f(x) = \frac{x^2-1}{x^2+3} \qquad f'(x) = \frac{8x}{(x^2+3)^2} \qquad f''(x) = \frac{-24(x^2-1)}{(x^2+3)^3}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: $(-\infty, \infty)$

B. (1 point) y-intercept: $f(0) = \frac{0^2-1}{0^2+3} = -\frac{1}{3}$

C. (1 point) x-intercept(s): $f(x)=0 \iff x^2-1=0 \iff x=\pm 1$

D. (1 point) Horizontal asymptote(s): $y=1$

$\lim_{x\to\infty} \frac{x^2-1}{x^2+3} = 1 \qquad \lim_{x\to-\infty} \frac{x^2-1}{x^2+3} = 1$

E. (1 point) Interval(s) $f(x)$ is increasing: $(0,\infty)$

F. (1 point) Interval(s) $f(x)$ is decreasing: $(-\infty, 0)$

$f'(x)$ is alway defined

$f'(x)=0$ when $x=0$

$f(x)$: $\searrow$ $\nearrow$

sign of $f'(x)$: $-$ $+$ (at 0)

G. (1 point) Local maximum(s) (x,y): none

H. (1 point) Local minimum(s) (x,y): $(0, f(0)) = (0, -\frac{1}{3})$

I. (1 point) Interval(s) $f(x)$ is concave up: $(-1, 1)$

J. (1 point) Interval(s) $f(x)$ is concave down: $(-\infty,-1)$, $(1,\infty)$

$f''(x)$ is alway defined

$f''(x)=0$ when $x=\pm 1$

$f(x)$: CD CU CD

Sign of $f''(x)$: $-$ $+$ $-$ (at -1, 1)

K. (1 point) Inflection point(s) (x,y): $(-1, f(-1)) = (-1, 0)$ $\quad (1, f(1)) = (1, 0)$

L. (5 points) Sketch $y = f(x)$ on the graph below.