Name Solutions Rec. Instr.

Signature Rec. Time

Math 220
Exam 3
November 21, 2019

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		25	5		10
2		12	6		8
3		10	7		14
4		10	8		11

Total Score	

1. (5 points each) Evaluate the following:

A. $$\lim_{\theta\to 0}\frac{\theta^2+\theta^3}{\sin(\theta)} \underset{\left(\substack{LH\\ \frac{0}{0}}\right)}{=} \lim_{\theta\to 0}\frac{2\theta+3\theta^2}{\cos(\theta)} = \frac{2\cdot 0+3\cdot 0^2}{\cos(0)} = 0$$

B. $$\lim_{x\to\infty}\frac{3x+e^x}{5x^2+7} \underset{\left(\substack{LH\\ \frac{\infty}{\infty}}\right)}{=} \lim_{x\to\infty}\frac{3+e^x}{10x} \underset{\left(\substack{LH\\ \frac{\infty}{\infty}}\right)}{=} \lim_{x\to\infty}\frac{e^x}{10} = \infty$$

C. $$\int\left(\frac{1}{x^2}+3\cos(x)\right)dx = -\frac{1}{x}+3\sin(x)+C$$

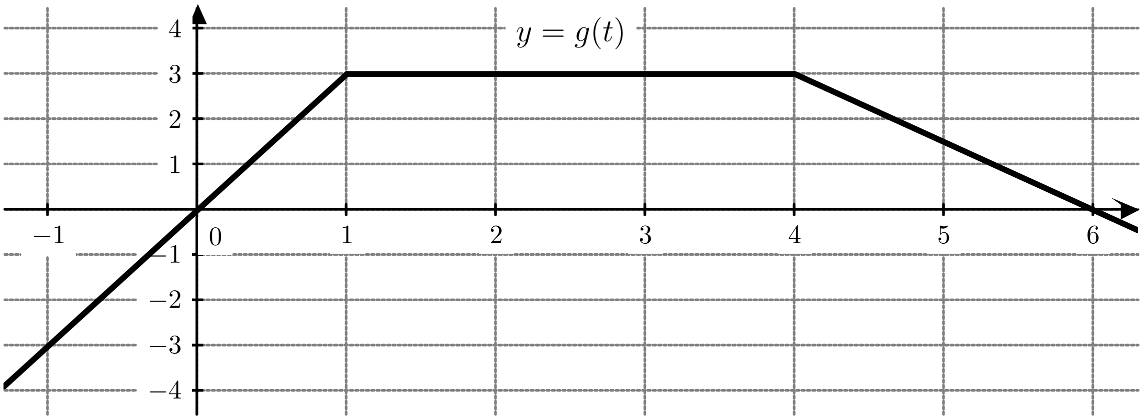
D. $$\frac{d}{dx}\int_x^5 3t^7\cos(t^9)\,dt = -\frac{d}{dx}\int_5^x 3t^7\cos(t^9)\,dt = -3x^7\cos(x^9)$$

E. $$\int_0^\pi (2\sin(\theta)+1)\,d\theta = \left(-2\cos(\theta)+\theta\right)\Big|_0^\pi$$

$$= \left(-2\cos(\pi)+\pi\right)-\left(-2\cos(0)+0\right)$$

$$= (2+\pi)-(-2)$$

$$= 4+\pi$$



2. (4 points each) $y = g(t)$ is plotted above. Let $A(x) = \int_0^x g(t)\,dt$. Find the following quantities.

A. $\int_{-1}^{1} g(t)\,dt = -\frac{1}{2}\cdot 1\cdot 3 + \frac{1}{2}\cdot 1\cdot 3 = 0$

B. $\int_{4}^{2} g(t)\,dt = -\int_2^4 g(t)dt = -2\cdot 3 = -6$

C. $A'(2) = g(2) = 3$ because $A'(x) = \frac{d}{dx}\int_0^x g(t)dt = g(x)$.

3. (10 points) Find $f(x)$ if $f''(x) = e^x + 2$, $f'(0) = 7$, and $f(0) = -5$.

$f'(x) = e^x + 2x + C$ for some constant C

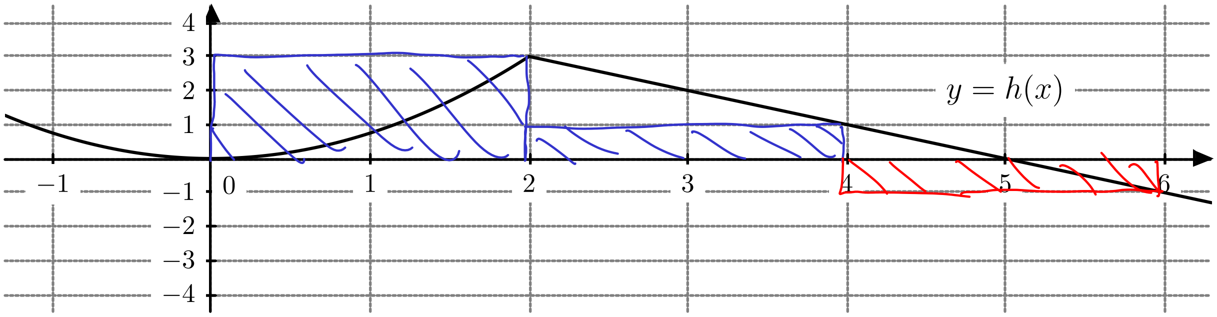
$7 = f'(0) = e^0 + 2\cdot 0 + C = 1 + C$ so $C = 6$

$f'(x) = e^x + 2x + 6$

$f(x) = e^x + x^2 + 6x + D$ for some constant D

$-5 = f(0) = e^0 + 0^2 + 6\cdot 0 + D = 1 + D$ so $D = -6$

$f(x) = e^x + x^2 + 6x - 6$



4. (10 points) Estimate $\int_0^6 h(x)\,dx$ by computing R_3, the Right-Endpoint Approximation with 3 subintervals. Also, illustrate the rectangles on the graph above.

$$\Delta x = \frac{6-3}{3} = 2$$

$$\int_0^6 h(x)dx \approx R(3) = f(2)\cdot\Delta x + f(4)\Delta x + f(6)\Delta x$$

$$= 3\cdot 2 + 1\cdot 2 - 1\cdot 2 = 6$$

5. The cost in dollars of producing x pounds of a chemical in a factory is given by $C(x)$. Suppose that the "marginal cost" is $C'(x) = \frac{x}{2} + 1$ \$/pound.

A. (7 points) Find $\int_2^4 C'(x)\,dx$. (Include units with your answer.)

$$\int_2^4 C'(x)\,dx = \int_2^4 \left(\frac{x}{2}+1\right)dx = \left(\frac{x^2}{4}+x\right)\Big|_2^4$$

$$= \left(\frac{4^2}{4}+4\right) - \left(\frac{2^2}{4}+2\right)$$

$$= 8-3 = \$5$$

B. (3 points) What does $\int_2^4 C'(x)\,dx$ represent? $\int_2^4 C'(x)dx = C(4) - C(2)$.

It is the cost to raise production from 2 pounds to 4 pounds of the chemical (the total cost of making the third and fourth pound).

6. (8 points) Suppose that a particle has position $s(t)$ feet at time t seconds and a velocity function $s'(t) = 3t^2 - 2t$ ft/s. If $s(0) = 5$ ft, find $s(2)$. (Include units with your answer.)

$$S(2)-s(0)=\int_0^2 s'(t)\,dt=\int_0^2 (3t^2-2t)\,dt$$

$$=(t^3-t^2)\Big|_0^2=(2^3-2^2)-(0^3-0^2)=4$$

$$S(2)=s(0)+4=5+4=9 \text{ ft.}$$

7. (7 points each) Evaluate the following:

A. $\displaystyle\int_0^{\pi/2} e^{2\sin(\theta)}\cos(\theta)\,d\theta = \int_0^2 e^u\cdot\frac{du}{2} = \frac{e^u}{2}\Big|_0^2 = \frac{e^2}{2}-\frac{e^0}{2}=\frac{e^2-1}{2}$

$u=2\sin(\theta)$

$du=2\cos(\theta)d\theta$

$\frac{du}{2}=\cos(\theta)d\theta$

θ	u
$\frac{\pi}{2}$	2
0	0

B. $\displaystyle\int x\sqrt{x-2}\,dx = \int (u+2)\sqrt{u}\,du = \int (u^{3/2}+2u^{1/2})\,du$

$$= \frac{2}{5}u^{5/2}+\frac{4}{3}u^{3/2}+C$$

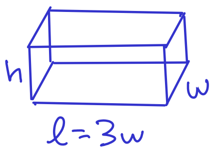
$$= \frac{2}{5}(x-2)^{5/2}+\frac{4}{3}(x-2)^{3/2}+C$$

$u=x-2$

$u+2=x$

$du=dx$

8. (11 points) A rectangular **open-topped** box is to have volume 18 m^3. The length of the base is to be **three times its width**. What dimensions minimize the amount of material needed to make the box? (Justify why your answer corresponds to an absolute minimum. Include units with your answer.)



Minimize area $A = 3w\cdot w + 2\cdot 3wh + 2\cdot wh = 3w^2 + 8wh$

$18 = 3w\cdot w\cdot h = 3w^2h$ so $h = \frac{18}{3w^2} = \frac{6}{w^2}$

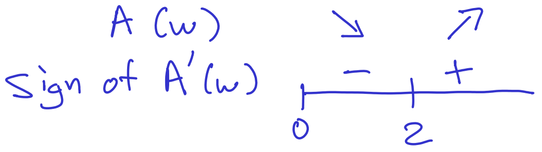
Minimize $A(w) = 3w^2 + 8wh = 3w^2 + 8w\cdot\frac{6}{w^2} = 3w^2 + \frac{48}{w}$ on $(0,\infty)$

$A'(w) = 6w - \frac{48}{w^2}$ is defined on $(0,\infty)$.

$0 = A'(w) = 6w - \frac{48}{w^2} \Rightarrow 6w = \frac{48}{w^2} \Rightarrow 6w^3 = 48$

$\Rightarrow w^3 = 8 \Rightarrow w = 2$

1st Derivative Justification



2nd Derivative Justification

$A''(w) = 6 + \frac{96}{w^3} > 0$ on $(0,\infty)$

The amount of material is minimized when the dimensions are

$w = 2$ m
$l = 3w = 3\cdot 2 = 6$ m
$h = \frac{6}{w^2} = \frac{6}{2^2} = \frac{6}{4} = \frac{3}{2}$ m