NAME ______________________ Rec. Instructor: ______________

Signature ______________________ Rec. Time ______________

| pg 1 | pg 2 | pg 3 | pg 4 | pg 5 | pg 6 | pg 7 | pg 8 | pg 9 | pg 10 | Total |
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| 15 | 25 | 25 | 25 | 24 | 23 | 12 | 17 | 20 | 14 | 200 |

# CALCULUS I - FINAL EXAM - SPRING 2019
May 15, 2019

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. No need to simplify derivatives or integrals. You can use guess and check for integrals.

(8) 1. Find the equation of the line tangent to $y = 2x^3 - x^2 + 4$ at $x = 1$.

(7) 2. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $x^3 + xy^3 = 7y + 3$.

**(25)** 3. Differentiate the following ( $\arctan x$ is the inverse tangent $\tan^{-1} x$ ).

a) $\frac{d}{dx}\left(3^x + 5\arctan x\right) =$

b) $\frac{d}{dx}\left(3\sqrt{x} + \frac{2}{x^5}\right) =$

c) $\frac{d}{dx}\left(x^3 \tan x\right) =$

d) $\frac{d}{dx}\left(\frac{x}{x^3+1}\right) =$

e) $\frac{d}{dx}\left(\int_2^{x^2} \sqrt{1+t^2}\,dt\right) =$

**(25)** 4. Differentiate the following with the chain rule

a) $\frac{d}{dx}\left(\ln(x^2+5)\right) =$

b) $\frac{d}{dx}\left(\cos^7(3x+5)\right) =$

c) $\frac{d}{dx}\left(e^{\sin x}\right) =$

d) $\frac{d}{dx}\left(\frac{1}{(x^2+7x-1)^3}\right) =$

e) $\frac{d}{dx}\left(\sqrt{1+e^{2x}}\right) =$

**(25)** 5. Integrate the following:

a) $\int \frac{1}{\sqrt{4-x^2}} + \sec^2 x \, dx =$

b) $\int \frac{u^2+2}{u^3} \, du =$

c) $\int (3t+1)^{10} \, dt =$

d) $\int \cos^5 x \ \sin x \, dx =$

e) $\int x^2 e^{-x^3} \, dx =$

(24) 6. Integrate the following:

a) $\int \frac{t^2}{1+t^3}\,dt =$

b) $\int \frac{1}{x\,(\ln x)^3}\,dx =$

c) $\int_0^4 \sqrt{1+2x}\,dx =$

d) $\int x(x+2)^{99}\,dx =$

**(15)** 7. Evaluate the limits:

a) $\lim\limits_{x \to 2} \dfrac{x^2 - 2x}{x^2 - 4} =$

b) $\lim\limits_{x \to \infty} \dfrac{x^2 - 5x^3 + 6}{x^3 + x - 9} =$

c) $\lim\limits_{x \to 0} \dfrac{1 + 2x - e^{2x}}{x^2} =$

**(8)** 8. The radius of a circular oil spill is increasing at a rate of 3 feet per minute. At what rate is the area increasing when the radius of the circle is 20 feet?

9. The function $f$ and its first and second derivatives are given:

$$f(x) = \frac{3(x^2-1)}{x^2+3}, \qquad f'(x) = \frac{24x}{(x^2+3)^2}, \qquad f''(x) = \frac{-72(x^2-1)}{(x^2+3)^3},$$

Obtain the following information.

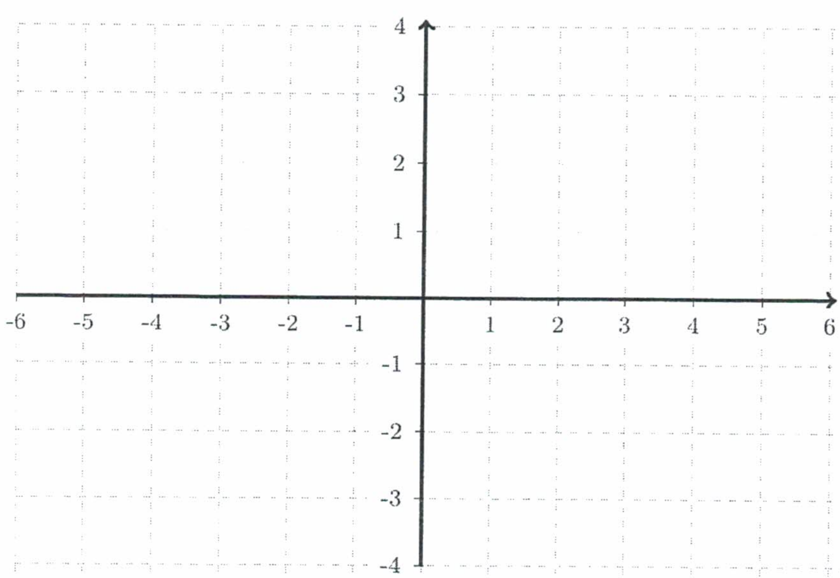
**(2)** (a) The intercepts:____________________.

**(2)** (b) Horizontal asymptotes: __________. Vertical asymptotes: NONE.

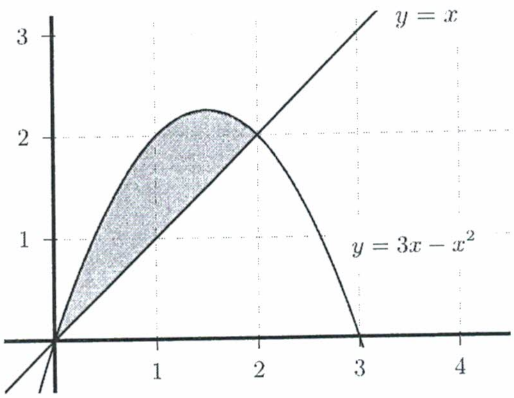
**(3)** (c) Coordinates of local maxima: ____________, local minima: ____________.

**(3)** (d) Coordinates of the inflection points: ________________________________________.

**(2)** (e) Use this information to sketch the graph of $f$



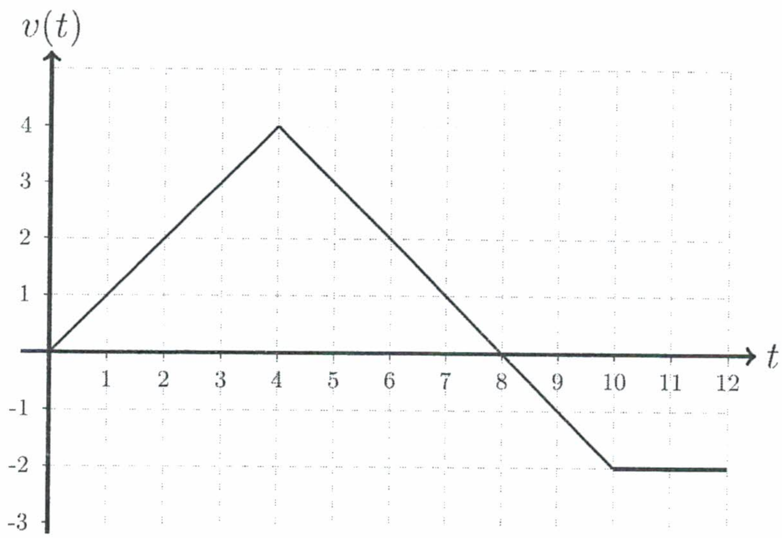
(7) 10. a) Find the area of the region between the curves $y = x$ and $y = 3x - x^2$




(5) b) Use the disc/washer method to set up an integral representing the volume obtained by rotating the region in (a) about the $x$-axis. Do **not** evaluate it!

(5) c) Use the cylindrical shell method to set up an integral representing the volume obtained by rotating the region in (a) about the $y$-axis. Do **not** evaluate it!

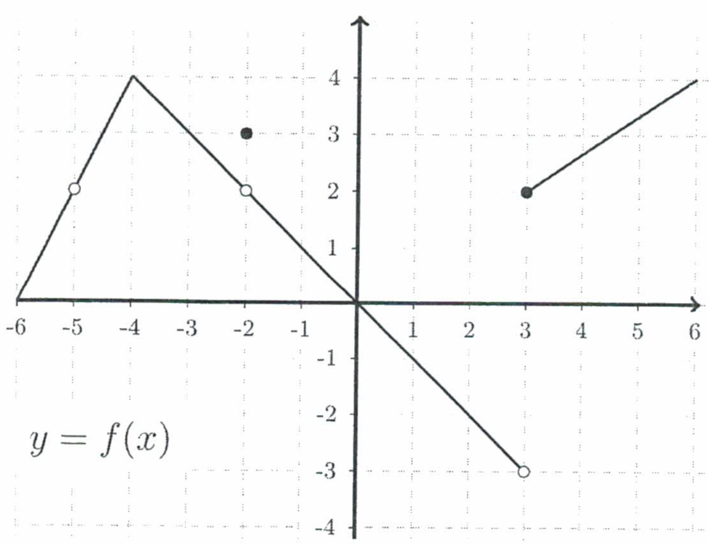
**(8)** 11. A particle moves forwards and backwards in a straight line for 12 seconds. The graph of its velocity $v(t)$ is shown in ft/sec.




a) What is the distance between its starting and finishing points? ______________.

b) What was the total distance that the particle travelled? ______________.

**(12)** 12. Use the graph to find the following. Put *does not exist* if appropriate.




a) $\lim_{x \to -2} f(x) =$ ________

b) $\lim_{x \to 3^+} f(x) =$ ________

c) $f'(0) =$ ________

d) $\int_0^2 f(x)\, dx =$ ________

(7) 13. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for $y = (x+1)^x$

(7) 14. Give the limit definition of the derivative of a function $f(x)$ at $x = a$

$$f'(a) = \lim \quad \underline{\hspace{4cm}}$$

and use it to find $f'(2)$ for $f(x) = 3x^2$.