

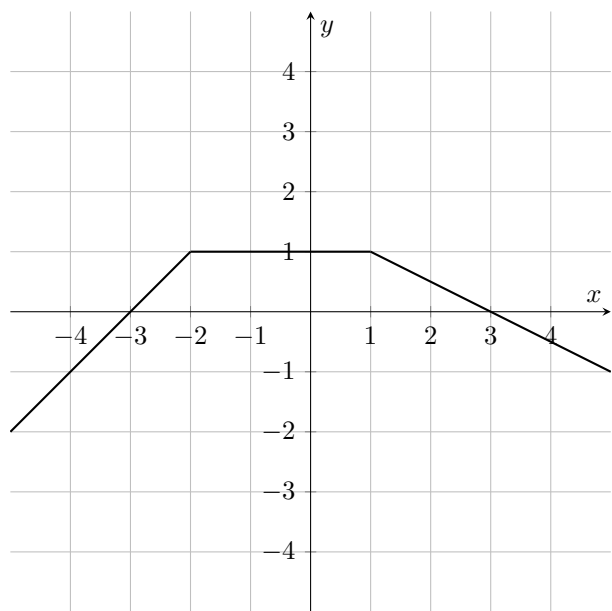
Name:\_\_\_\_\_ Instructor:\_\_\_\_\_

Signature:\_\_\_\_\_ Wildcat ID:\_\_\_\_\_

Math 220  
Exam 2  
July 3, 2019

No books, calculators, or notes are allowed. Please make sure your cell phone is turned off. You will have 75 minutes to complete the exam.

| Problem     | Points | Points Possible |
|-------------|--------|-----------------|
| 1           |        | 8               |
| 2           |        | 8               |
| 3           |        | 4               |
| 4           |        | 4               |
| 5           |        | 15              |
| 6           |        | 12              |
| 7           |        | 8               |
| 8           |        | 5               |
| 9           |        | 8               |
| 10          |        | 6               |
| 11          |        | 8               |
| 12          |        | 6               |
| 13          |        | 8               |
| 14          |        | 5 (EC)          |
| Total Score |        | 100             |



1. (2 points each) Consider the graph of  $y = f(x)$  above. State the value of each of the quantities below. If the derivative does not exist at the specified point (i.e., if  $f(x)$  is not differentiable at the specified point), write “does not exist.” You do not need to show any work.

(a)  $f'(-3) =$

(b)  $f'(-2) =$

(c)  $f'(-1) =$

(d)  $f'(2) =$

2. (8 points) Use the **definition of the derivative** to find  $f'(1)$ , where  $f(x) = \sqrt{x+3}$ . Show all work (if you simply use derivative rules, such as the power rule, instead of using the definition of the derivative, you will receive no credit).

3. (4 points) Calculate  $f'(x)$ , where  $f(x) = \frac{1}{x^2} + \sqrt[3]{x}$

4. (4 points) Calculate  $f'(x)$  and  $f''(x)$ , where  $f(x) = 4x^3 - \sin(x)$ .

5. (5 points each) Calculate  $\frac{dy}{dx}$  for the each of the following. You do not need to simplify your answers.

(a)  $y = 2x^4 \tan^{-1}(x)$

(b)  $y = \frac{\ln(x)}{\tan(x)}$

(c)  $y = e^{\cos(x^2)}$

**6.** (6 points each)

(a) Find the equation of the tangent line to the graph of  $f(x) = x^4 - x^2 + 3$  at  $x = 1$ .

(b) Find the equation of the tangent line to the graph of  $g(x) = \sin(2x)$  at  $x = \frac{\pi}{2}$ .

7. (8 points) Let  $f(x) = 2x^2 - 4x + 1$  for  $x \geq 1$ . Use the Inverse Function Theorem to find  $(f^{-1})'(1)$ .

8. (5 points) Let  $f(x)$  be a differentiable function, with  $f(3) = -2$  and  $f'(3) = 6$ . Find  $(f^{-1})'(-2)$ .

9. (8 points) Use logarithmic differentiation to find  $f'(x)$ , where  $f(x) = x^{3x^2}$ . You do not need to simplify your answer.

**10.** (6 points) Use implicit differentiation to find  $y'$ , if  $\sin(xy) + y^2 = 2x$ .

**11.** (8 points) Use implicit differentiation to find the tangent line to the curve  $2y^3 + 5y - x^3 = -x + 1$  at the point  $(2, 1)$ .

**12.** (6 points) Use L'Hôpital's Rule to evaluate  $\lim_{x \rightarrow \infty} \frac{2x^2}{e^{3x}}$

**13.** (8 points each) Use L'Hôpital's Rule to evaluate  $\lim_{x \rightarrow 0^+} x \ln(x)$ . Show all work.

14. (Extra Credit: 5 points) Use implicit differentiation or the Inverse Function Theorem to show that

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}.$$

Show all work in order to receive credit. No partial credit will be awarded on this problem.