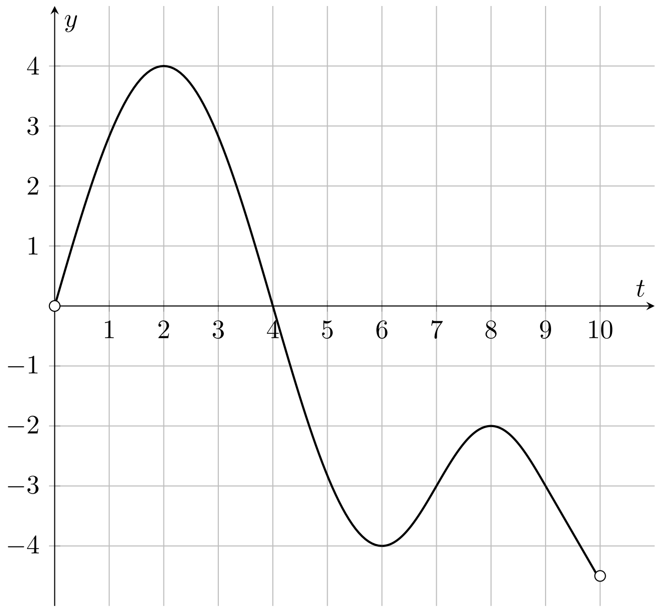
Name:____________________ Instructor:____________________

Signature:____________________ Wildcat ID:____________________

Math 220
Exam 3
July 18, 2019

No books, calculators, or notes are allowed. Please make sure your cell phone is turned off. You will have 75 minutes to complete the exam.

Problem	Points	Points Possible
1		8
2		10
3		15
4		12
5		14
6		26
7		15
8		5 (EC)
Total Score		100



1. The graph above shows the position $y = s(t)$ of a particle traveling along a horizontal line for time $t \in (0, 10)$.

(a) (6 points) Determine the times when the velocity is zero, and the time intervals when the velocity is positive or negative. You do not need to show any work.

- zero:

- positive:

- negative:

(b) (2 points) Is the acceleration positive, negative, or zero on the time interval $(0, 4)$? You do not need to show any work.

2. (10 points) Use linearization to approximate $\sqrt{9.1}$. You can leave your answer as a whole number plus or minus a fraction (e.g., if you get $7 + \frac{1}{8}$, you do not need to write it as 7.125).

3. (15 points) A 12-m ladder is leaning against a wall. If the bottom of the ladder is 3 m from the wall at time $t = 0$, and the bottom of the ladder slides away from the wall at a rate of 1 m/s, how fast is the top of the ladder moving down the wall after 3 seconds (i.e., what is the **speed** of the top of the ladder after 3 seconds)? Include units in your final answer.

4. (a) (4 points) Find all critical points of the function $f(x) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 + 1$.

(b) (8 points) Use the **First Derivative Test** to classify each critical point of $f(x)$ as a local minimum, local maximum, or neither. Show all of your work **clearly**. You will receive no credit if you use a method other than the First Derivative Test (e.g., the Second Derivative Test).

5. (14 points) Find where all of the absolute extrema of $f(x)$ occur for $f(x) = \cos(x) + x$ in $[0, 2\pi]$, and classify each as a minimum or a maximum. Show all work clearly in order to receive full credit.

6. Let $f(x) = \frac{1}{3}x^3 - x^2$.

(a) (2 points) Find the zeros of $f(x)$.

(b) (5 points) Find all the transition points of $f(x)$.

(c) (2 points) Evaluate $f(x)$ at all transition points (unless $f(x)$ is undefined at a transition point, in which case, write "undefined").

(d) (6 points) Use the **Second Derivative Test** to classify each critical point of $f(x)$ as a local minimum, local maximum, or inconclusive. Show all of your work **clearly**. You will receive no credit if you use a method other than the Second Derivative Test (e.g., the First Derivative Test).

(e) (3 points) List the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

(f) (3 points) List the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.

(g) (5 points) Using all of the information you found in parts (a)-(e), sketch a graph of $f(x)$. In order to receive full credit, make sure your graph is drawn and labeled clearly enough to show that you used all of the information from parts (a)-(e).

7. (15 points) A rancher has 400 m of fencing and wants to build a rectangular pen of maximum area next to a river (i.e., one side of the rectangle doesn't need fencing). Find the dimensions that maximize the area inside the fence.

8. (Extra Credit: 5 points) No partial credit will be awarded on this problem.

Definition: We say the function $f(x)$ is increasing on the interval I if, given any two points a and b in I with $a < b$,

$$f(a) \leq f(b).$$

Theorem: Let $f(x)$ be differentiable on the interval I. If $f'(x) > 0$ for all $x \in I$, then $f(x)$ is increasing on I.

In class, we learned this theorem but did not prove it. Here, we will begin a proof, and you will have to finish the proof. The proof will use a common technique called proof by contradiction. In this technique, we assume that the conclusion we want is false, and we show that this leads to a contradiction, which cannot occur. Therefore, our extra assumption cannot be false, so it must be true.

A contradiction is the logical term for a statement that is opposed to itself, i.e., a statement that is both true and false. For example, "$a < 0$ and $a \geq 0$." These two things cannot occurs simultaneously for the same a, so the statement "$a < 0$ and $a \geq 0$" is a contradiction.

Proof. Let a and b be two arbitrary points in I, with $a < b$. We wish to show $f(a) \leq f(b)$, thereby showing that $f(x)$ is increasing on I.

We will assume that that $f(a)$ is not less than or equal to $f(b)$, i.e., $f(a) > f(b)$, and we will try to derive a contradiction from this.

Since our hypotheses about the function $f(x)$ are that it is differentiable on I, and that $f'(x) > 0$ for all $x \in I$, we hope to contradict one of these two facts.

Your objective is to derive a contradiction, thereby finishing the proof. If you use a theorem that we have discussed in class, you must reference it by name in order to receive credit.

$\square$