MATH 220 - FINAL EXAM

Rec. instr.: _____

December 15th, 2021

Rec. time: _____

Use a **pen** to fill out this page. **Make sure your handwriting is legible and the print is dark enough for scanning purposes.**

First name (please write as legibly as possible within the boxes)

Last name

Wildcat ID (this number starts with 8 and is printed in your student card)

First and last names must be exactly the same as they appear in Canvas.

The solution of each problem **must** be written on the space provided in the page where the problem is stated. Solutions written on a different page/space will not be considered. Solutions must be written with a **pencil #2 or a pen. Make sure your handwriting is legible and the print is dark enough for scanning purposes.**

Show all work for full credit. No books, cell phones, notes, calculators or formula sheets are permitted. The point value of each problem is given in the left-hand margin.

1. **(9 pts)** Find an equation of the line tangent to $y = x^5 + 3x$ at $x = 1$. Show your work.

2. **(9 pts)** Use implicit differentiation to find $\dfrac{dy}{dx}$ if $y^5x^3 = 4x + y + 10$. Show your work.

3. (18 pts) Differentiate the following functions.

a) $\dfrac{d}{dx}\left(\arctan x + 2^x \ln x + \dfrac{1}{\sqrt[3]{x}}\right) =$

b) $\dfrac{d}{dx}\left(\dfrac{\sin x}{x^4+2}\right) =$

c) $\dfrac{d}{dx}\left(\displaystyle\int_0^{x^2} \cos(u+3)\, du\right) =$

4. (18 pts) Differentiate the following functions using the chain rule.

a) $\dfrac{d}{dx}\left(\tan(x^3+5x+1)\right) =$

b) $\dfrac{d}{dx}\left(\dfrac{1}{(x^2+3x+1)^5}\right) =$

c) $\dfrac{d}{dx}\left(e^{\cos(x)}\right) =$

5. (18 pts) Find the following integrals.

a) $\displaystyle\int \frac{1}{x^2+4} + 3\sin x + \frac{1}{x}\, dx =$

b) $\displaystyle\int \frac{u^2+3}{\sqrt{u}}\, du =$

c) $\displaystyle\int \cos x \ \sin^3 x \, dx =$

6. (18 pts) Find the following integrals. Show your work.

a) $\displaystyle\int \frac{(\ln x)^4}{x}\,dx =$

b) $\displaystyle\int 3t^2\sqrt{1+t^3}\,dt =$

c) $\displaystyle\int_{-2}^{-1} x\sqrt{x+2}\,dx =$

7. (18 pts) Evaluate the following limits. Show your work.

a) $\lim_{x \to 1} \dfrac{x^2 + 2x - 3}{x^2 + x - 2} =$

b) $\lim_{x \to 0} \dfrac{\sin x + x^2}{\cos x - 1 + 3x} =$

c) $\lim_{x \to -\infty} \dfrac{5x^4 - x + 1}{x^4 - 2x^2 + 3} =$

8. (10 pts) The radius of a circle increases at a rate of 2 m/sec. Find the rate at which the area of the circle increases when the radius is 5 m. Show your work.

Note: The area of a circle of radius r is πr^2.

9. (20 pts) The function f and its first and second derivatives are given:

$$f(x) = \frac{x}{(x^2+9)^2}, \quad f'(x) = \frac{3\,(3-x^2)}{(x^2+9)^3}, \quad f''(x) = \frac{12\,x\,(x^2-9)}{(x^2+9)^4}.$$

Obtain the following information, writing *none* when appropriate. Show your work.

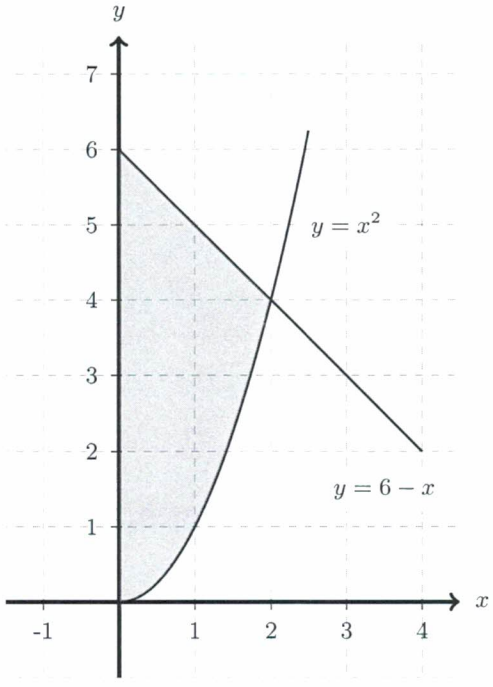
a) Where is f increasing? Where is f decreasing?

b) x-coordinate of local maxima and x-coordinate of local minima.

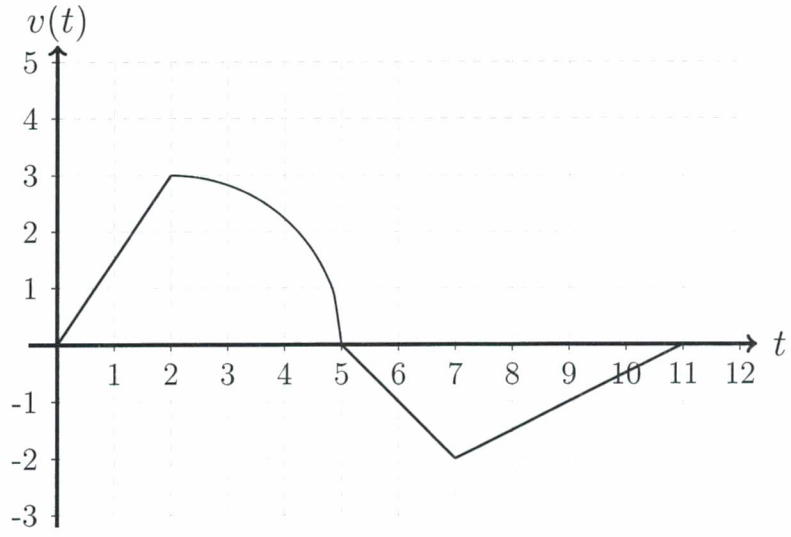
c) Where is f concave up? Where is f concave down?

d) x-coordinate of all inflection points.

10. (11 pts) Find the area of the region between the y-axis, the curve $y = 6 - x$ and the curve $y = x^2$. Show your work (you can use the picture to deduce the points of intersection of the curves; no need to show work for calculating them).



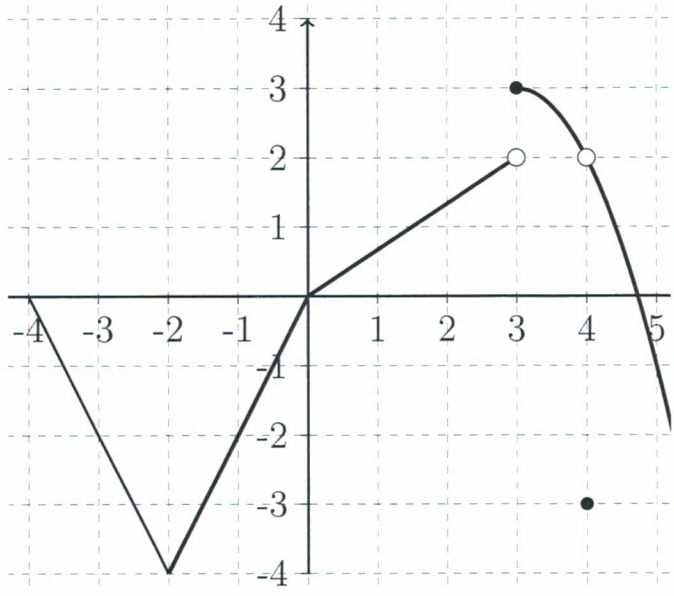
11. (10 pts) A car moves forwards and backwards in a straight line for 11 minutes. The graph of its velocity $v(t)$ is shown in ft/min. The curved part corresponding to $2 \leq t \leq 5$ is a fourth of a circle.



a) When was the car moving forwards, when backwards? Explain.

b) What is the distance between its starting and finishing points? Show your work.

12. (25 pts) Use the graph to find the following limits, derivative and integral. Write *does not exist* when appropriate.



a) $\lim_{x \to 4} f(x) =$

b) $\lim_{x \to 3^-} f(x) =$

c) $\lim_{x \to 3} f(x) =$

d) $f'(-1) =$

e) $\int_{-4}^{3} f(x)\, dx =$

13. (8 pts) Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y = \frac{(x-3)^x}{x^4}$. Show your work.

14. (8 pts) Use the limit definition of the derivative to find $f'(1)$ for $f(x) = x^2 + 5$. Show your work.