Math 220 Midterm 1

Name: _____

Recitation instructor: _____

Recitation time: _____

- This is a closed-book, closed-notes exam. No calculators or electronic aids are permitted.
- Read each question carefully and show your work.

Grading

1	/16	**2**	/8
3	/8	**4**	/10
5	/15	**6**	/23
7	/20	**Total**	100

Problem 1. (*16 points*) Evaluate the following limits.

A. (*3 points*) $\lim_{x \to 1} (x^6 + 4\sqrt{x} + 1)$

B. (*3 points*) $\lim_{\theta \to \pi/2} \dfrac{\sin(\theta)}{\theta}$

C. (*4 points*) $\lim_{t \to -1} \dfrac{t^2 + 3t + 2}{t + 1}$

D. (*6 points*) $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

Problem 2. (*8 points*) Use the Intermediate Value Theorem to show that there is a root of $f(x) = x^3 + 2x - 3$ in the interval $(0,2)$. **Make sure to mention any properties of $f(x)$ required to apply the Intermediate Value Theorem.**

Problem 3. (*8 points*) Given that $\lim_{x\to 5} u(x) = 1$ and $\lim_{x\to 5} w(x) = 3$, find the following limits.

A. (*4 points*) $\lim_{x\to 5} \dfrac{w(x) - 2}{u(x)}$

B. (*4 points*) $\lim_{x\to 5} \dfrac{\sqrt{u(x) \cdot w(x)}}{x^2 - 1}$

Problem 4. (*10 points*) Suppose that an object is at position $s(t) = \dfrac{2}{\sqrt{t}}$ feet at time t seconds.

A. (*4 points*) Find the average velocity of the object over a time interval from time 2 seconds to time $2 + h$ seconds.

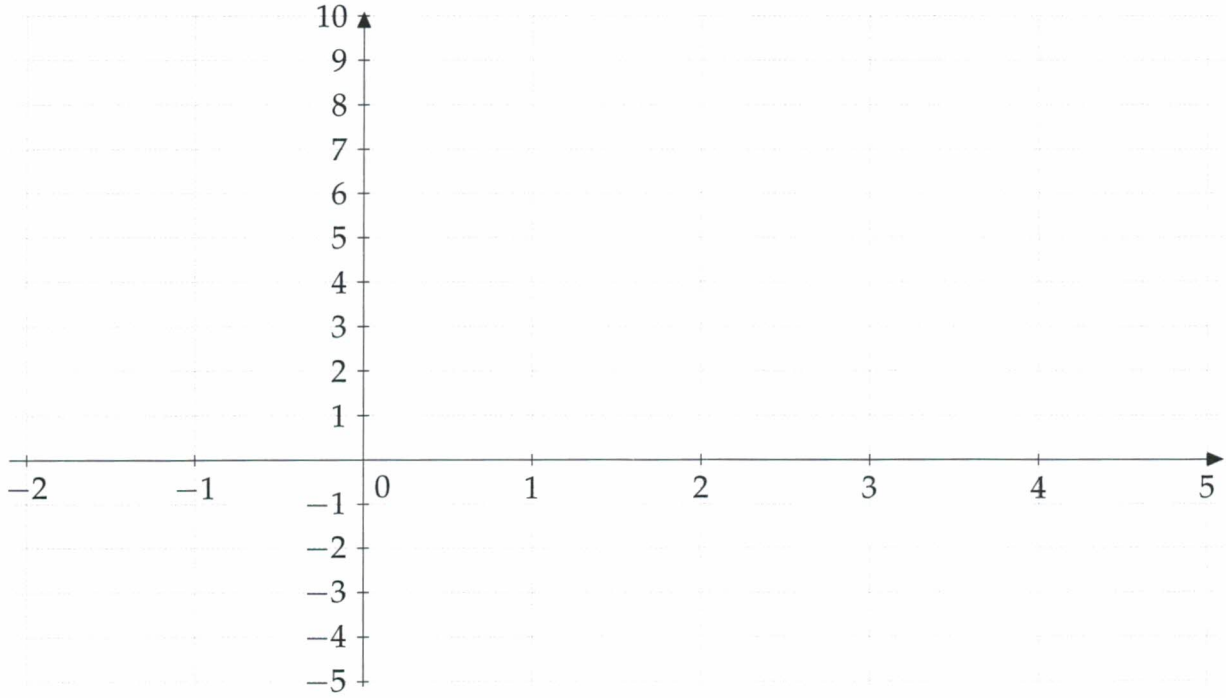
B. (*6 points*) Find the instantaneous velocity of the object at time 2 seconds by taking the limit of the average velocity in Part A as $h \to 0$.

Problem 5. (*15 points*) Let $v(x) = x^2 + 2x - 1$.

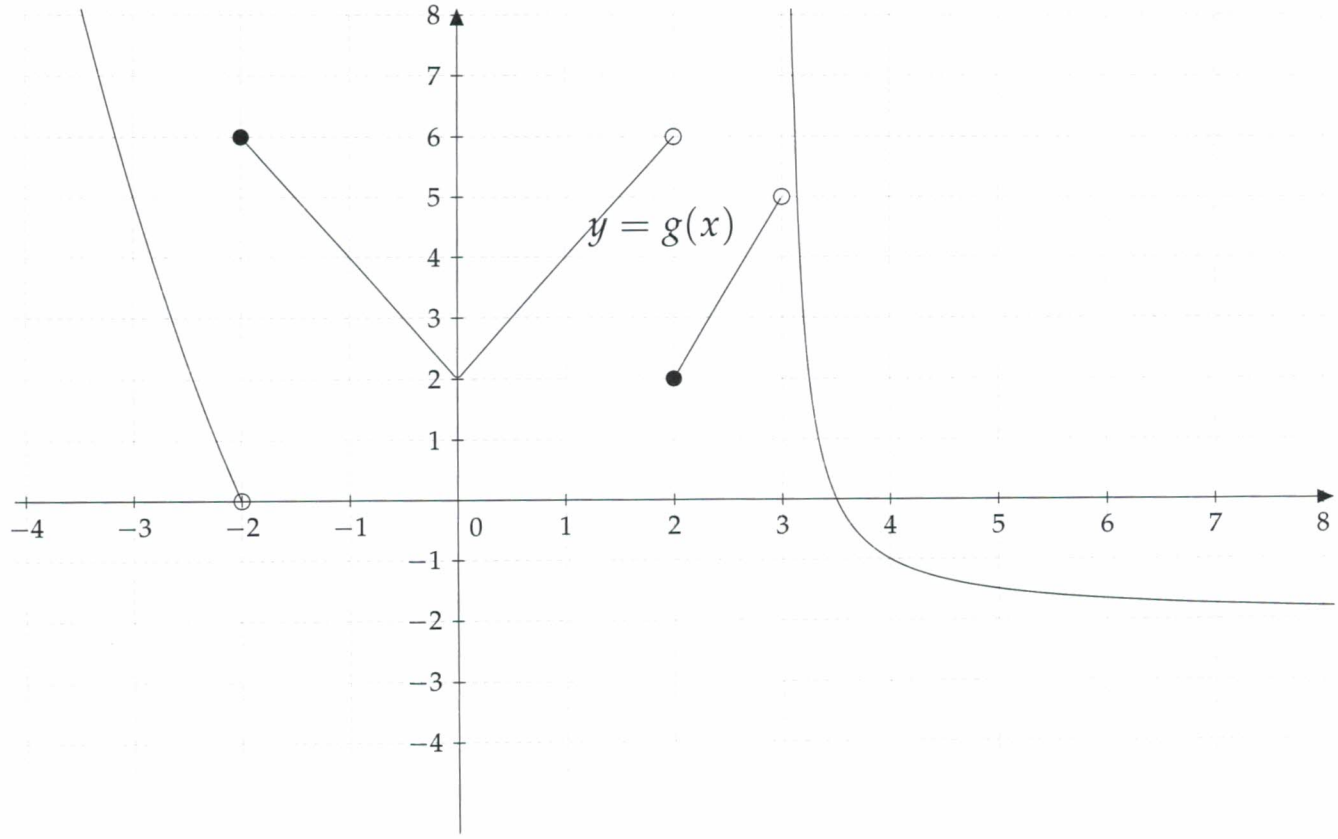
A. (*6 points*) Find the slope of the tangent line to $v(x)$ at $x = 0$ by using one of the limit definitions of the derivative.

B. (*4 points*) Find the equation of the tangent line to $y = v(x)$ at $x = 0$.

C. (*5 points*) Sketch $v(x)$ and the tangent line to $y = v(x)$ at $x = 0$ in the grid below.



Problem 6. (*23 points*)



(A - H: 2 points each) State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x \to -2^-} g(x)$

B. $\lim_{x \to -2^+} g(x)$

C. $g(-2)$

D. $g'(-2)$

E. $\lim_{x \to 2^-} g(x)$

F. $\lim_{x \to 2^+} g(x)$

G. $\lim_{x \to 2} g(x)$

H. $g'(-1)$

I. (3 points) x in $(-4, 8)$ at which $g(x)$ is discontinuous:

J. (4 points) x in $(-4, 8)$ at which $g'(x)$ is undefined:

Problem 7. (*20 points*) Find the following derivatives. **You do NOT need to simplify.**

A. (*4 points*) $\frac{d}{dx}(x^{10} + \sqrt{x})$

B. (*5 points*) (Use the Product Rule) $\frac{d}{dx}[(x^{1/3} + x^{-3})(x^2 - 1)]$

C. (*5 points*) (Use the Product Rule) $\frac{d}{dx}[(x^3-2)(x^{\pi}+x^{1/5})]$

D. (*6 points*) $\frac{d}{dx}\left(\frac{x^2+6x}{\sqrt{x}}\right)$