Math 220 Midterm 3

Name: _____

Recitation instructor: _____

Recitation time: _____

- This is a closed-book, closed-notes exam. No calculators or electronic aids are permitted.

- Read each question carefully and show your work unless explicitly told otherwise.

Grading

1	/15	2	/12
3	/17	4	/10
5	/15	6	/15
7	/16	**Total**	/100

Problem 1. (*15 points*) The function $f(x)$ and its first and second derivatives are:

$$f(x) = \sqrt{x^2 + 2x} \qquad f'(x) = \frac{x+1}{\sqrt{x^2 + 2x}} \qquad f''(x) = -\frac{1}{(x^2 + 2x)^{3/2}}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (3 point) Domain of $f(x)$: _____

B. (1 point) y-intercept: _____

C. (1 point) x-intercept(s): _____

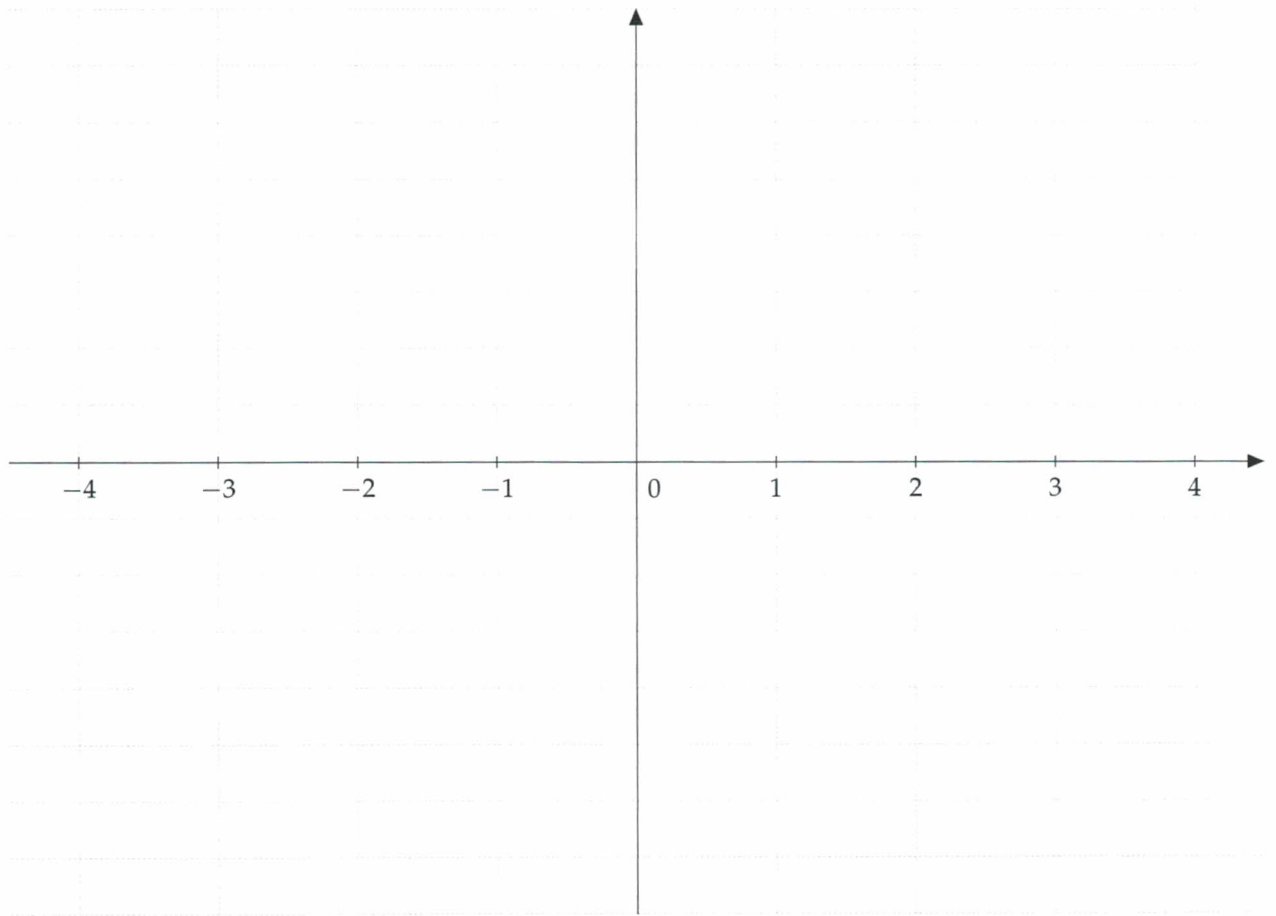**D.** (1 point) Interval(s) $f(x)$ is increasing: _____

E. (1 point) Interval(s) $f(x)$ is decreasing: _____

F. (1 point) Interval(s) $f(x)$ is concave up: _____

G. (1 point) Interval(s) $f(x)$ is concave down: _____

H. (1 point) Inflection point(s) (x, y): _____

I. (5 points) Sketch $y = f(x)$ on the graph below.

Problem 2. (*12 points*) (3 points each) In each of the following blanks, fill in "**local max**", "**absolute max**", "**local min**" or "**absolute min**".

A. If $h'(3) = 0$ and $h''(3) = 2$, then $h(x)$ has a _____ at $x = 3$.

B. If $h'(-2) = 0$ and $h''(-2) = -11$, then $h(x)$ has a _____ at $x = -2$.

C. If $f'(0) = 0$ and $f'(x) > 0$ on $(-\infty, 0)$ and $f'(x) < 0$ on $(0, \infty)$, then $f(x)$ has a

_____ at $x = 0$.

D. If $g'(-1) = 0$ and $g''(x) = -1$ for all real numbers x, then $g(x)$ has a

_____ at $x = -1$.

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Problem 3. (*17 points*) Find the following limits. (Use limit notation correctly.)

A. (*5 points*) $\displaystyle\lim_{x\to\infty} \frac{x}{e^x}$

B. (*5 points*) $\displaystyle\lim_{x\to-\infty} \frac{1+e^x}{\sqrt{e^x+1}}$

C. (*7 points*) $\lim\limits_{x \to 0^+} x^x$

Problem 4. (*10 points*) Let $f(x) = \dfrac{x\sqrt{x^2+9}}{x^2+3x+2}$.

A. (*4 points*) Find the vertical asymptote(s) of $f(x)$.

B. (*6 points*) Find the horizontal asymptote(s) of $f(x)$.

Problem 5. (*15 points*)   A rectangular open-topped aquarium is to have a square base and volume 9 m^3. The material for the base costs \$2 per m^2, and the material for the sides costs \$3 per m^2. What dimensions minimize the cost of the aquarium? (Make sure to justify why your answer corresponds to an absolute minimum.)

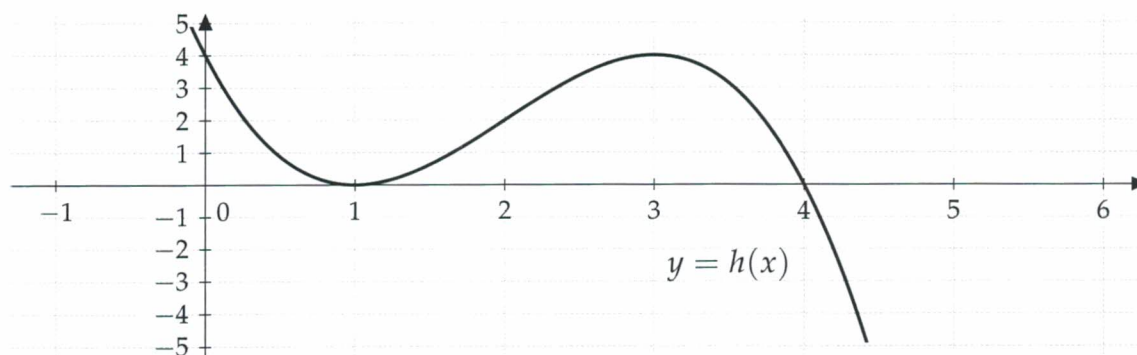**Problem 6.** (*15 points*) Evaluate the following indefinite integrals - you do not need to simplify.

A. (4 points) $\int 2x + 2x^{-2} + 3x^{2022} dx$

B. (5 points) $\int \sqrt{x} + \dfrac{5 + x^2}{1 + x^2} dx$

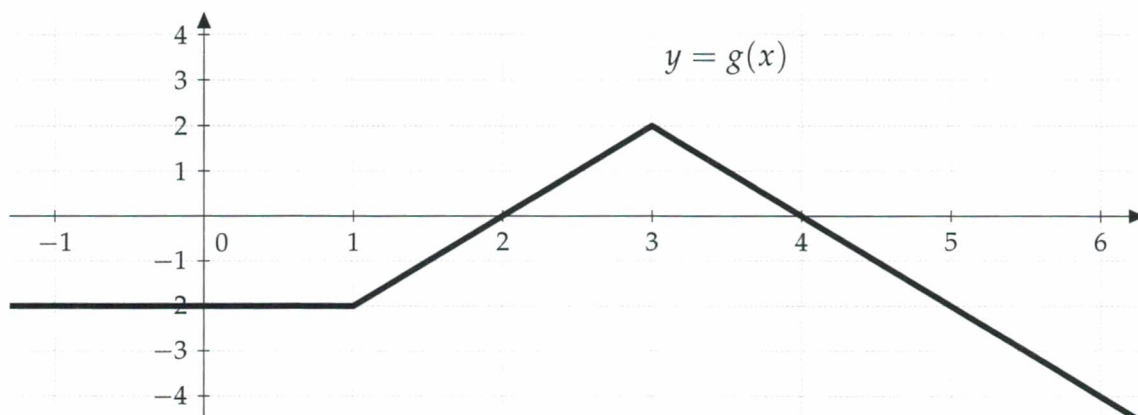**C.** (6 points) $\int e^x + \dfrac{1}{x} + \sec(x) \tan(x) dx$

Problem 7. (*16 points*)

A. (8 points)

$y = h(x)$ is plotted above. Estimate $\int_0^4 h(x)\,dx$ by using a Riemann sum with $n = 4$ subintervals, taking the sampling points to be right endpoints (the Right Hand Rule R_4). Also, illustrate the rectangles on the graph above.

B. (8 points)

$y = g(x)$ is plotted above. Evaluate the following definite integrals. No work needs to be shown.

i. $\displaystyle\int_{1}^{2} g(x)\,dx =$

ii. $\displaystyle\int_{1}^{0} g(x)\,dx =$

iii. $\displaystyle\int_{2}^{6} g(x)\,dx =$

iv. $\displaystyle\int_{1}^{3} g(x)\,dx =$