Name Solutions Rec. Instr.

Signature Rec. Time

Math 220

Exam 1

February 3, 2022

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

SHOW YOUR WORK!

Problem	Points	Points Possible	Problem	Points	Points Possible
1		12	7		12
2		7	8		4
3		4	9		9
4		8	10		9
5		8	11		16
6		7	12		4

Total Score:

1. (4 points each) Evaluate the following limits.

A. $\lim_{x\to 3}(5+x^2)$ $= 5+3^2 = 14$

B. $\lim_{\theta\to 0}\frac{\cos(\theta)}{1-\theta}$ $= \frac{\cos(0)}{1-0} = 1$

C. $\lim_{\theta\to 0}\frac{7\sin(\theta)}{3\theta}$ $= \frac{7}{3}\left(\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}\right) = \frac{7}{3}$

2. (7 points) Use the Intermediate Value Theorem to show that there is a root of $f(x) = e^x + x - 2$ in the interval $(0, 2)$. (**Make sure to mention any properties of $f(x)$ required to apply the Intermediate Value Theorem.**)

$f(x)$ is continuous on $[0,2]$.

$f(0) = e^0 + 0 - 2 = -1$

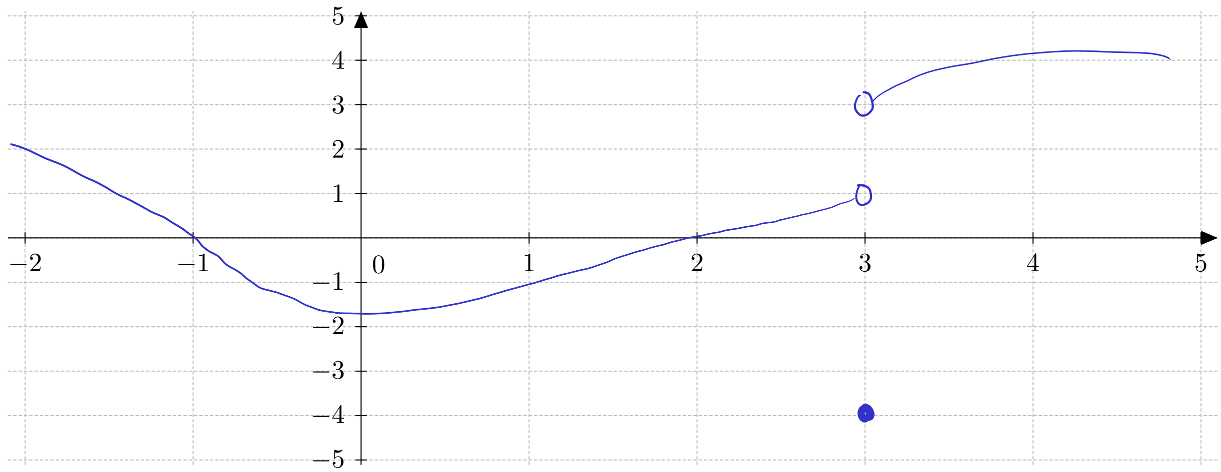
$f(2) = e^2 + 2 - 2 = e^2$

By the Intermediate Value Theorem, there exists a number c in $(0,2)$ with $f(c) = 0$.

3. (4 points) Find functions $b(x)$ and $c(x)$ such that $b(c(x)) = \tan(x^2 + 1)$.

$b(x) =$ $\tan(x)$ $\qquad$ $c(x) =$ x^2+1

4. (8 points) Sketch the graph of a function $k(x)$ that satisfies $\lim_{x\to -1} k(x) = 0$, $\lim_{x\to 3^-} k(x) = 1$, $\lim_{x\to 3^+} k(x) = 3$, and $k(3) = -4$.



5. (4 points each) Given that $\lim_{x\to -2} u(x) = 4$ and $\lim_{x\to -2} w(x) = 5$, find the following limits.

A. $\lim_{x\to -2} \dfrac{3u(x)}{w(x)}$ $= \dfrac{3\cdot 4}{5} = \dfrac{12}{5}$

B. $\lim_{x\to -2} \dfrac{\sqrt{w(x)+4}}{x}$ $= \dfrac{\sqrt{5+4}}{-2} = -\dfrac{3}{2}$

6. (7 points) Find $\lim_{x\to 2} m(x)$ provided that the function $m(x)$ satisfies $5x - 4 \le m(x) \le x^2 + x$ for all $x \neq 2$. (**Justify your reasoning, and state the name of any theorem used.**)

$\lim_{x\to 2} (5x-4) = 5\cdot 2 - 4 = 6$

$\lim_{x\to 2} (x^2+x) = 2^2+2 = 6$

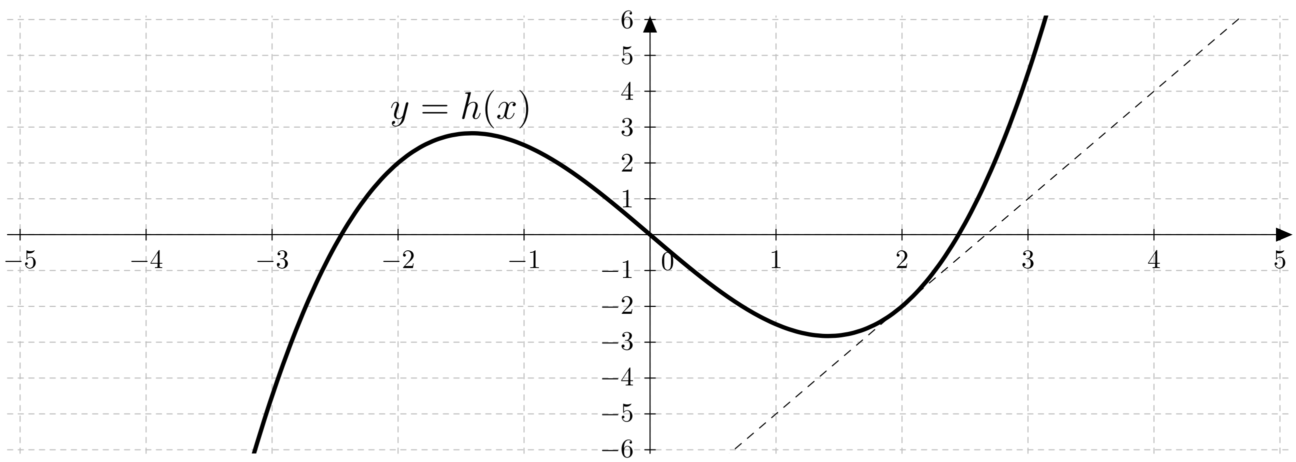
By the Squeeze Theorem, $\lim_{x\to 2} m(x) = 6$.

7. (6 points each) Evaluate the following limits.

A. $\lim_{w\to 3} \frac{w^2-9}{w-3}$ $= \lim_{w\to 3} \frac{\cancel{(w-3)}(w+3)}{\cancel{w-3}} = \lim_{w\to 3} (w+3) = 3+3 = 6$

B. $\lim_{x\to 4} \frac{2-\sqrt{x}}{4-x}$ $= \lim_{x\to 4} \frac{2-\sqrt{x}}{4-x} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} = \lim_{x\to 4} \frac{4+\cancel{2\sqrt{x}}-\cancel{2\sqrt{x}}-x}{(4-x)(2+\sqrt{x})}$

$= \lim_{x\to 4} \frac{\cancel{4-x}}{\cancel{(4-x)}(2+\sqrt{x})} = \lim_{x\to 4} \frac{1}{2+\sqrt{x}} = \frac{1}{2+\sqrt{4}} = \frac{1}{4}$



8. (2 points each) The function $y = h(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values.

A. $h(2) = -2$

B. $h'(2) = 3$

9. Let $v(x) = \dfrac{2}{x}$.

A. (6 points) **Using the limit definition of the derivative**, find $v'(1)$.

$$v'(1) = \lim_{h\to 0} \frac{v(1+h)-v(1)}{h} = \lim_{h\to 0} \frac{\frac{2}{1+h} - \frac{2}{1}}{h} = \lim_{h\to 0} \frac{2}{h(1+h)} - \frac{2}{h}$$

$$= \lim_{h\to 0} \frac{2}{h(1+h)} - \frac{2(1+h)}{h(1+h)} = \lim_{h\to 0} \frac{\cancel{2}-\cancel{2}-2h}{h(1+h)} = \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}(1+h)}$$

$$= \lim_{h\to 0} \frac{-2}{1+h} = \frac{-2}{1+0} = -2$$

B. (3 points) Find the equation of the tangent line to $y = v(x)$ at $x = 1$.

$v(1) = \frac{2}{1} = 2$ so $y - 2 = -2(x-1)$

(or $y = -2x + 4$)

10. Suppose that an object is at position $s(t) = t^2 + 3$ feet at time t seconds.

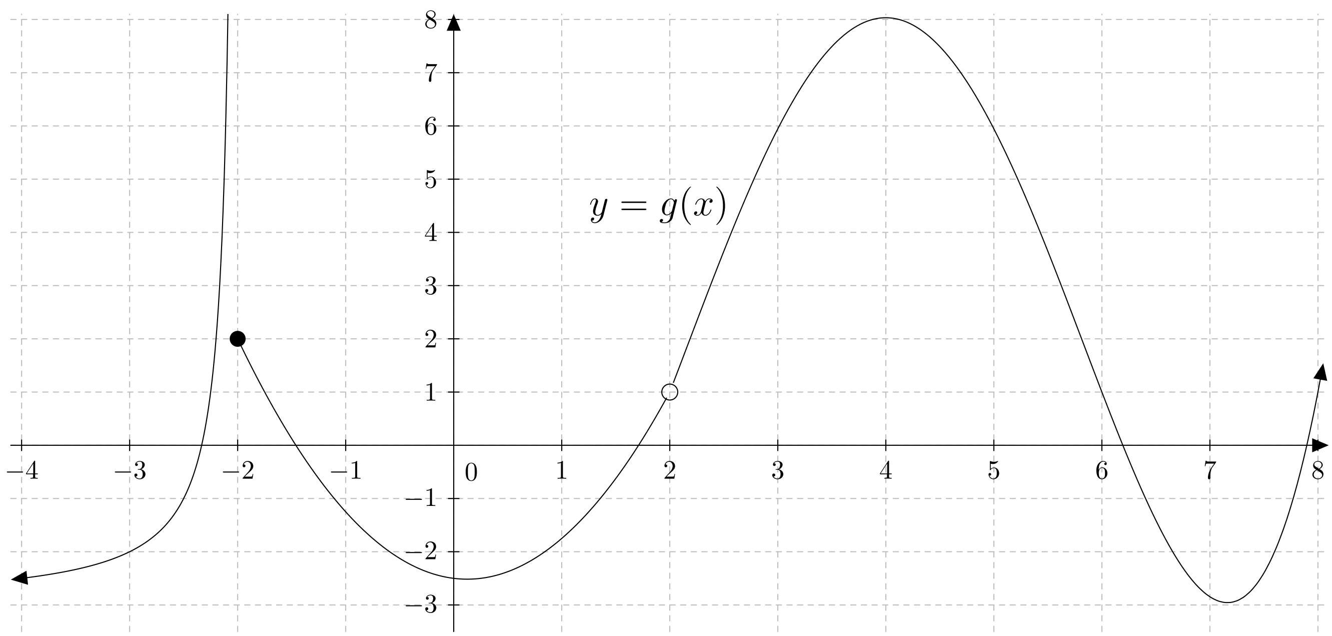
A. (3 points) Find the average velocity of the object over a time interval from time 2 seconds to time $2 + h$ seconds.

$$\frac{s(2+h) - s(2)}{(2+h) - 2} = \frac{((2+h)^2 + \cancel{3}) - (2^2 + \cancel{3})}{h} = \frac{(2+h)^2 - 4}{h} \text{ ft/s}$$

B. (6 points) Find the instantaneous velocity of the object at time 2 seconds by taking the limit of the average velocity in Part A as $h \to 0$.

$$\lim_{h\to 0} \frac{(2+h)^2 - 4}{h} = \lim_{h\to 0} \frac{\cancel{4} + 4h + h^2 - \cancel{4}}{h} = \lim_{h\to 0} \frac{4h + h^2}{h}$$

$$= \lim_{h\to 0} (4+h) = 4 \text{ ft/s}$$



11. (2 points each) Consider the graph of $y = g(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x \to -2^-} g(x) = \infty$

B. $\lim_{x \to -2^+} g(x) = 2$

C. $\lim_{x \to -2} g(x)$ does not exist

D. $g(-2) = 2$

E. $\lim_{x \to 2^-} g(x) = 1$

F. $\lim_{x \to 2^+} g(x) = 1$

G. $\lim_{x \to 2} g(x) = 1$

H. $g(2)$ does not exist

12. (4 points) Consider the graph of $y = g(x)$ above. List the x-coordinates where the function is discontinuous.

$x = -2$ and $x = 2$