Name Solutions Rec. Instr.
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Math 220
Exam 2
March 3, 2022

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		30	6		5
2		10	7		10
3		9	8		9
4		9	9		9
5		9	**Total Score**		100

1. (6 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\dfrac{d}{dx}\left(\dfrac{3}{x} + 2^x + \ln(3)\right)$ $= -\dfrac{3}{x^2} + 2^x \ln(2)$

B. $\dfrac{d}{dx}\left(x^{3/2} \cdot \arctan(x)\right)$ $= \dfrac{3}{2}\sqrt{x} \cdot \arctan(x) + x^{3/2} \cdot \dfrac{1}{1+x^2}$

C. $\dfrac{d}{dt}\tan(t^3 - 4t)$ $= \sec^2(t^3-4t)\cdot(3t^2-4)$

D. $\dfrac{d}{d\theta}\ln\left(\sin(2\theta)\right)$ $= \dfrac{1}{\sin(2\theta)} \cdot \cos(2\theta)\cdot 2$

E. $\dfrac{d}{dx}\left(\dfrac{\cos(x) - e^x}{e^x + 2}\right)$ $= \dfrac{(-\sin(x)-e^x)\cdot(e^x+2) - (\cos(x)-e^x)e^x}{(e^x+2)^2}$

2. (10 points) Find the equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

$$\frac{d}{dx}\sqrt{x}\Big|_{x=4} = \frac{1}{2\sqrt{x}}\Big|_{x=4} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$y - \sqrt{4} = \frac{1}{4}(x-4)$$

$$y - 2 = \frac{1}{4}(x-4) \qquad \left(\text{or} \quad y = \frac{1}{4}x + 1\right)$$

3. (9 points) Let $p(x) = x^2 - 3x + 1$. Using the **limit definition of the derivative**, find $p'(x)$. Make sure to use limit notation correctly.

$$p'(x) = \lim_{h\to 0} \frac{p(x+h) - p(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) + \cancel{1} - x^2 + 3x - \cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h} = \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h}$$

$$= \lim_{h\to 0} (2x + h - 3) = 2x + 0 - 3 = 2x - 3$$

4. (9 points) Find $\dfrac{dy}{dx}$ if $x^2y - x = e^y + 1$.

$$\frac{d}{dx}\left(x^2y - x\right) = \frac{d}{dx}\left(e^y + 1\right)$$

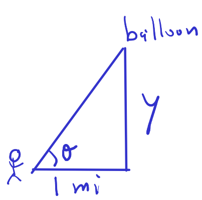
$$2xy + x^2\frac{dy}{dx} - 1 = e^y\frac{dy}{dx}$$

$$x^2\frac{dy}{dx} - e^y\frac{dy}{dx} = 1 - 2xy$$

$$(x^2 - e^y)\frac{dy}{dx} = 1 - 2xy$$

$$\frac{dy}{dx} = \frac{1-2xy}{x^2 - e^y}$$

5. (9 points) A hot air balloon rising vertically is tracked by an observer located 1 miles from the lift-off point. At a certain moment, the angle between the observer's line-of-sight and the horizontal is $\dfrac{\pi}{4}$, and the angle is changing at a rate of $\dfrac{1}{8}$ rad/min. How fast is the balloon rising at this moment?



Know: $\dfrac{d\theta}{dt} = \dfrac{1}{8}$ rad/min when $\theta = \dfrac{\pi}{4}$

Want: $\dfrac{dy}{dt}$ when $\theta = \dfrac{\pi}{4}$

$$\frac{y}{1} = \tan(\theta)$$

$$y = \tan(\theta)$$

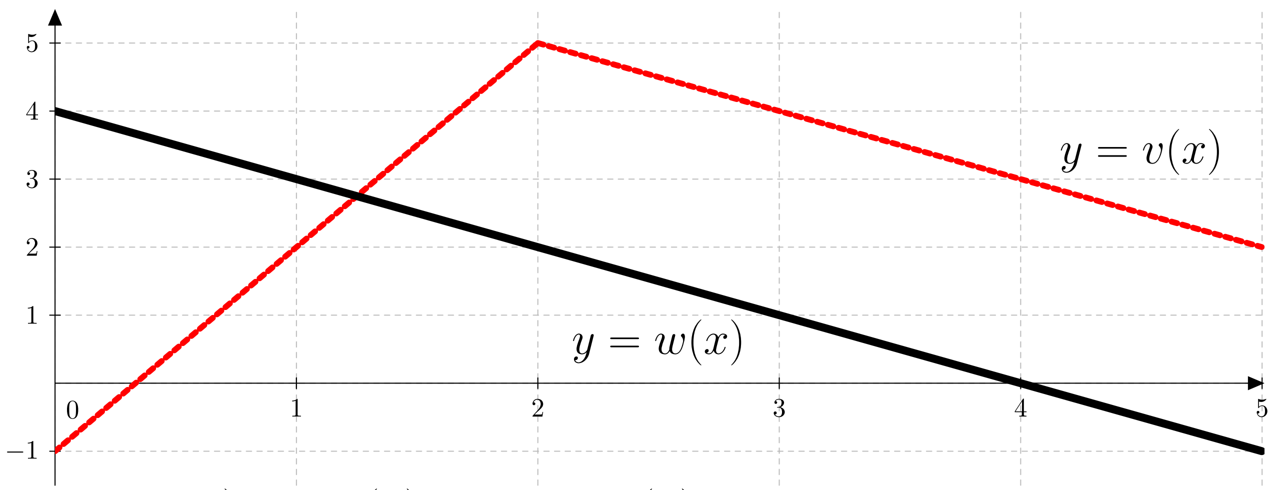
$$\frac{dy}{dt} = \frac{d}{dt}\tan(\theta) = \sec^2(\theta)\,\frac{d\theta}{dt}$$

When $\theta = \frac{\pi}{4}$, $\quad \dfrac{dy}{dt} = \sec^2\left(\dfrac{\pi}{4}\right)\cdot\dfrac{1}{8} = \dfrac{1}{\cos^2\left(\frac{\pi}{4}\right)}\cdot\dfrac{1}{8}$

$$= \frac{1}{\left(1/\sqrt{2}\right)^2}\cdot\frac{1}{8} = \frac{2}{8} = \frac{1}{4}\ \text{mi/min}$$

6. (5 points) Suppose that a waiter brings you a glass of cold lemonade on a hot day. Let $F'(t)$ denote the temperature in degrees Fahrenheit of the lemonade after t minutes. Is $F'(2)$ positive or negative? Explain your answer.

Positive, the lemonade is warming up two minutes after you get it.



7. (5 points each) $y = v(x)$ and $y = w(x)$ are graphed above. Suppose that $f(x) = v(x) \cdot w(x)$ and $g(x) = v(w(x))$. Find:

A. $f'(1)$ $= v'(1)w(1) + v(1)w'(1) = 3 \cdot 3 + 2 \cdot (-1) = 7$

B. $g'(3)$ $= v'(w(3))w'(3) = v'(1) \cdot (-1) = 3 \cdot (-1) = -3$

8. (9 points) Find the derivative of $h(x) = x^{8x}$.

$$\ln(h(x)) = \ln(x^{8x}) = 8x\ln(x)$$

$$\frac{d}{dx}\ln(h(x)) = \frac{d}{dx}(8x\ln(x))$$

$$\frac{h'(x)}{h(x)} = 8\ln(x) + 8x\cdot\frac{1}{x} = 8\ln(x)+8$$

$$h'(x) = h(x)(8\ln(x)+8) = x^{8x}(8\ln(x)+8)$$

9. (9 points) The volume of a sphere of radius r is $V = \dfrac{4}{3}\pi r^3$. If the radius of a sphere is 2 cm and increasing at a rate of $\frac{1}{2}$ cm/sec, find the rate at which the volume of the sphere is increasing.

Know: $\frac{dr}{dt} = \frac{1}{2}$ cm/s when $r = 2$cm

Want: $\frac{dV}{dt}$ when $r = 2$cm.

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 \frac{dr}{dt}$$

When $r = 2$cm, $\frac{dV}{dt} = 4\pi\cdot 2^2\cdot\frac{1}{2} = 8\pi$ cm^3/s.