Name Solutions ______________________ Rec. Instr. ______________________

Signature ______________________ Rec. Time ______________________

Math 220

Exam 3

April 7, 2022

7:05-8:20 PM

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		15	6		10
2		12	7		20
3		8	8		10
4		5	9		10
5		10	Total Score		100

1. The function $f(x)$ and its first and second derivatives are:

$$f(x) = x^2(x-3) \qquad f'(x) = 3x(x-2) \qquad f''(x) = 6(x-1).$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: $(-\infty, \infty)$

B. (1 point) y-intercept: 0

C. (1 point) x-intercept(s): 0 and 3

D. (1 point) Interval(s) $f(x)$ is increasing: $(-\infty, 0)$ and $(2, \infty)$

E. (1 point) Interval(s) $f(x)$ is decreasing: $(0, 2)$

$f'(x)$ is always defined and continuous.

$f'(x) = 0$ when $x = 0$ or 2

$f(x)$: ↗ ↘ ↗

sign of $f'(x)$: $+$ | $-$ | $+$

0 local max, 2 local min

F. (1 point) Local maximum(s) (x, y): $(0, 0)$

G. (1 point) Local minimum(s) (x, y): $(2, -4)$

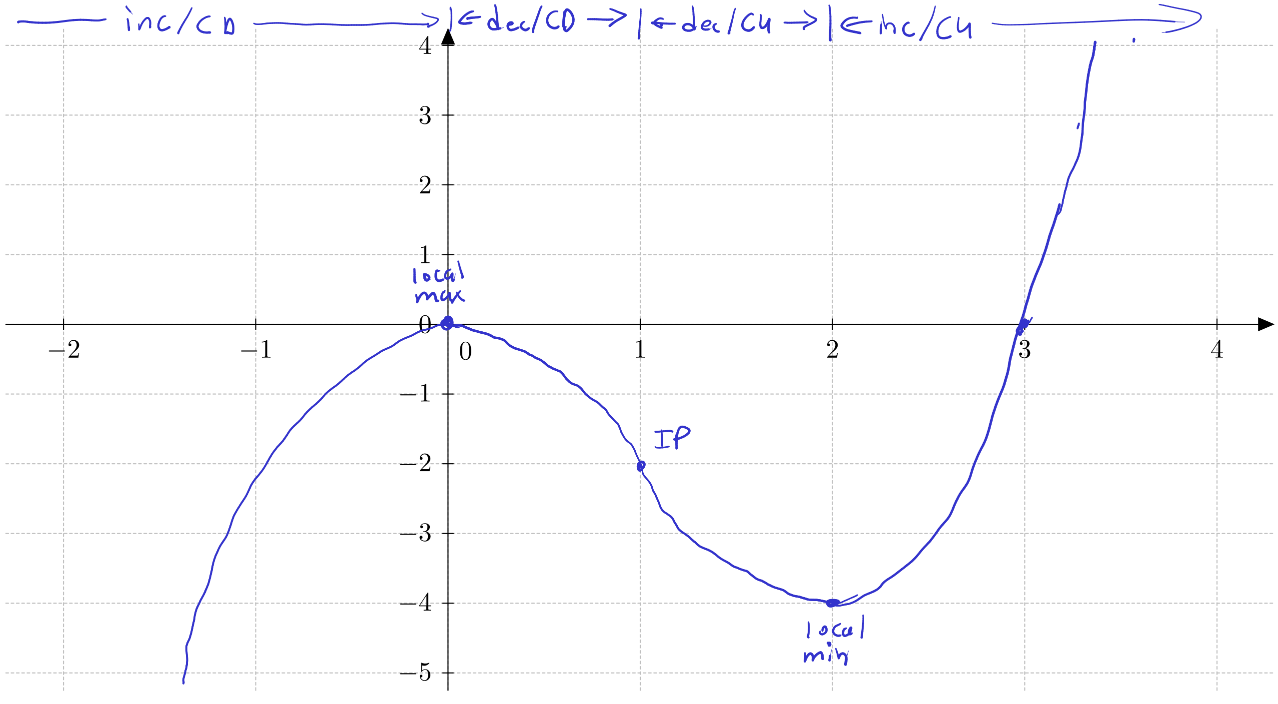
H. (1 point) Interval(s) $f(x)$ is concave up: $(1, \infty)$

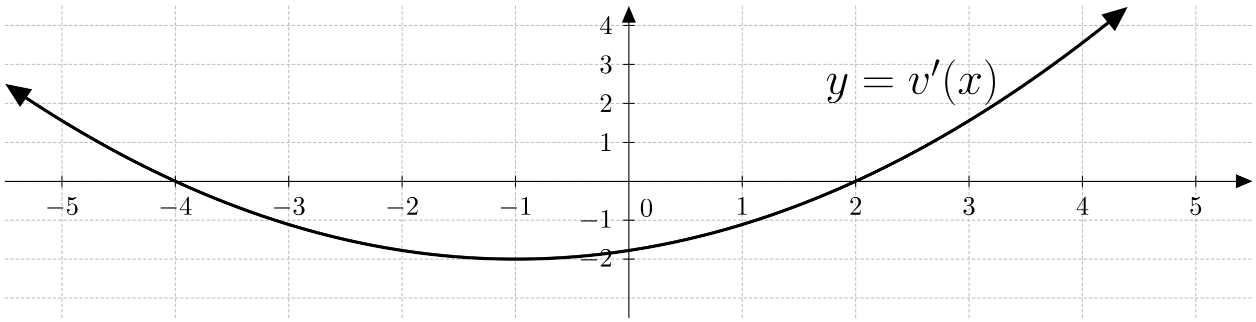
I. (1 point) Interval(s) $f(x)$ is concave down: $(-\infty, 1)$

J. (1 point) Inflection point(s) (x, y): $(1, -2)$

K. (5 points) Sketch $y = f(x)$ on the graph below.

inc/CD → | ← dec/CD → | ← dec/CU → | ← inc/CU →





2. (3 points each) $y = v'(x)$ is plotted above. Find:

A. Interval(s) where $v(x)$ is increasing: $(-\infty, -4)$ and $(2, \infty)$ decreasing: $(-4, 2)$

B. x-coordinate(s) where $v(x)$ has a local max: -4 local min: 2

C. Interval(s) where $v(x)$ is concave up: $(-1, \infty)$ concave down: $(-\infty, -1)$

D. x-coordinate(s) where $v(x)$ has an inflection point: -1

3. (4 points each) In each of the following blanks, fill in "**max**" or "**min**".

A. If $h'(4) = 0$ and $h''(4) = 2$, then $h(x)$ has a local min at $x = 4$.

B. If $h'(-3) = 0$ and $h''(-3) = -2.7$, then $h(x)$ has a local max at $x = -3$.

4. (5 points) Find the differential dy if $y = \cos(x^2 + 3)$.

$$\frac{dy}{dx} = -\sin(x^2+3) \cdot 2x$$

$$dy = -\sin(x^2+3) \cdot 2x \cdot dx$$

5. (10 points) Find the absolute maximum and absolute minimum of $w(x) = x^3 - 3x^2 + 1$ on $[-1, 1]$.

$w'(x) = 3x^2 - 6x = 3x(x-2)$ is always defined.

$w'(x) = 0$ at $x=0$ and $x=2$.

The only critical number of $w(x)$ in $[-1,1]$ is at $x=0$.

$w(-1) = (-1)^3 - 3\cdot(-1)^2 + 1 = -3$

$w(0) = 0^3 - 3\cdot 0^2 + 1 = 1$

$w(1) = 1^3 - 3\cdot 1^2 + 1 = -1$

On $[-1,1]$ $w(x)$ has an absolute max at $(0,1)$ and an absolute min at $(-1,-3)$.

6. A. (7 points) Find the linearization of $g(x) = \sin(x)$ at $x = 0$.

$g'(x) = \cos(x)$. The linearization of $g(x)$ at $x=0$ is

$L(x) = g(0) + g'(0)(x-0) = \sin(0) + \cos(0)\cdot x = x$

B. (3 points) Use your answer from Part **A** to estimate $\sin(.01)$.

$\sin(.01) = g(.01) \underset{\uparrow}{\approx} L(.01) = .01$

$\boxed{.01 \text{ is close to } 0}$

7. (5 points each) Find the following limits. (Use limit notation correctly.)

A. $\displaystyle\lim_{x\to\infty} \frac{e^x+2}{x^2+4} \underset{LH}{=} \lim_{x\to\infty} \frac{e^x}{2x} \underset{LH}{=} \lim_{x\to\infty} \frac{e^x}{2} = \infty$

(type $\frac{\infty}{\infty}$) (type $\frac{\infty}{\infty}$)

B. $\displaystyle\lim_{x\to 1} \frac{\ln(x)}{x-1} \underset{LH}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{1} = \lim_{x\to 1} \frac{1}{x} = \frac{1}{1} = 1$

(type $\frac{0}{0}$)

C. $\displaystyle\lim_{x\to -\infty} \frac{-5x^4+x+2}{7x^4-x^3-2x+1} = \frac{-5}{7}$

D. $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2+4}}{2x+1} = \lim_{x\to\infty} \frac{\frac{\sqrt{x^2+4}}{x}}{\frac{2x+1}{x}} = \lim_{x\to\infty} \frac{\frac{\sqrt{x^2+4}}{\sqrt{x^2}}}{2+\frac{1}{x}}$

$\boxed{x>0 \text{ so } x=\sqrt{x^2}}$

$\displaystyle= \lim_{x\to\infty} \frac{\sqrt{1+\frac{4}{x^2}}}{2+\frac{1}{x}} = \frac{\sqrt{1+0}}{2+0} = \frac{1}{2}$

8. (10 points) Suppose that when your bakery sells cakes for x dollars each, your total profit is $P(x) = -x^2 + 20x - 75$ dollars. In order to maximize total profit, how much should your bakery charge per cake? (Make sure to justify why your answer corresponds to the absolute maximum.)

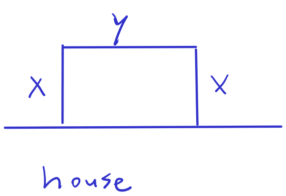
$P'(x) = -2x + 20$ is always defined

$P'(x) = 0 \iff 2x = 20 \iff x = 10$

1st Deriv. Justification		2nd Deriv Justification
$P(x)$: $\nearrow$ $\searrow$ sign of $P'(x)$: $+$ on $(0, 10)$, $-$ after 10	Or	$P''(x) = -2$ $P(x)$ is always concave down

$P(x)$ has an absolute maximum when you charge $x = \$10$ per cake.

9. (10 points) A homeowner with 20 feet of fencing wants to enclose a rectangular area against the side of her house. What dimensions will maximize the fenced-in area? (Note that three sides of the rectangle will be formed from fencing, and the house will serve as the fourth side of the rectangle. Make sure to justify why your answer corresponds to the absolute maximum.)



Maximize Area $A = xy$

$2x + y = 20$ ft so $y = 20 - 2x$

$A(x) = x(20 - 2x) = 20x - 2x^2$

$A'(x) = 20 - 4x$ is always defined

$A'(x) = 0 \iff 4x = 20 \iff x = 5$

1st Deriv. Justification		2nd Deriv. Justification		Closed Interval Method
$A(x)$: $\nearrow$ $\searrow$ sign of $A'(x)$: $+$ on $(0, 5)$, $-$ after 5	Or	$A''(x) = -4$ $A(x)$ is always concave down	Or	Neither x nor y can be negative. Since $y = 20 - 2x$, we must have that x is in $[0, 10]$. $A(0) = 20 \cdot 0 - 2 \cdot 0^2 = 0$ ft^2 $A(5) = 20 \cdot 5 - 2 \cdot 5^2 = 50$ ft^2 $A(10) = 20 \cdot 10 - 2 \cdot 10^2 = 0$ ft^2

The dimensions that maximize the area is when $x = 5$ ft and $y = 20 - 2 \cdot 5 = 10$ ft.