Name _____ Rec. Instr._____
Signature _____ Rec. Time_____

Math 220
Exam 3
April 7, 2022
7:05-8:20 PM

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		15	6		10
2		12	7		20
3		8	8		10
4		5	9		10
5		10	Total Score		100

1. The function $f(x)$ and its first and second derivatives are:

$$f(x) = x^2(x-3) \qquad f'(x) = 3x(x-2) \qquad f''(x) = 6(x-1).$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: _____

B. (1 point) y-intercept: _____

C. (1 point) x-intercept(s): _____

D. (1 point) Interval(s) $f(x)$ is increasing: _____

E. (1 point) Interval(s) $f(x)$ is decreasing: _____

F. (1 point) Local maximum(s) (x, y): _____

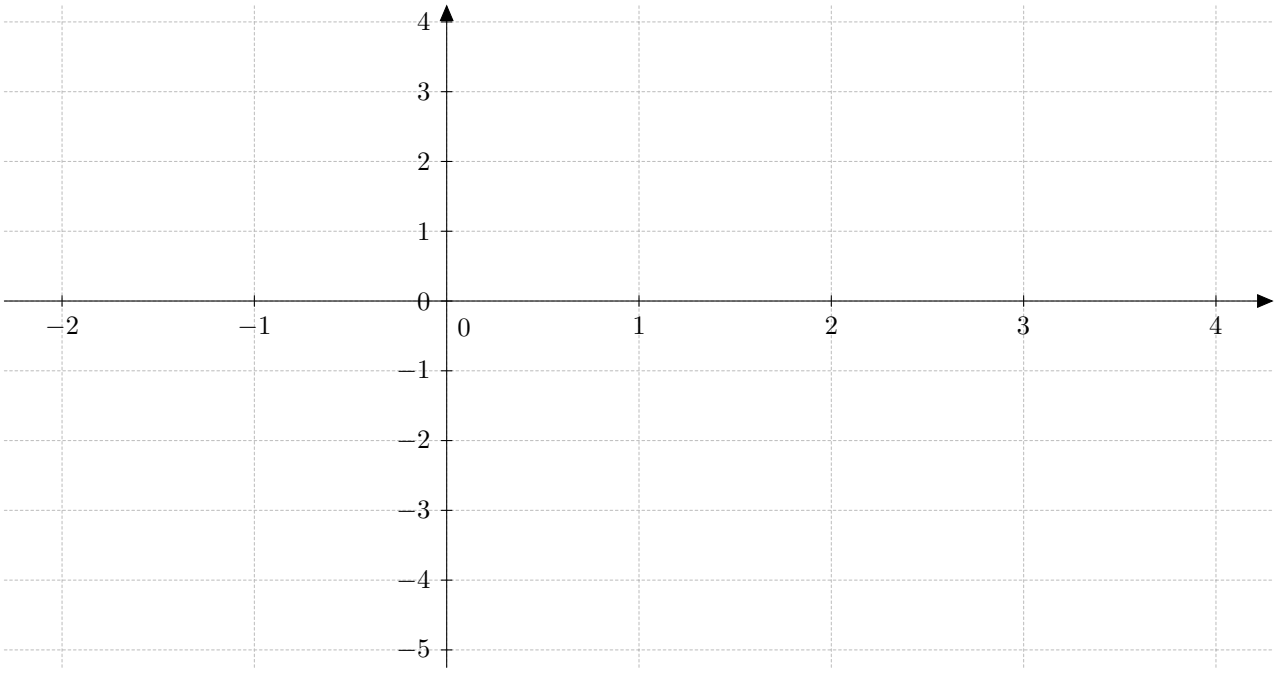
G. (1 point) Local minimum(s) (x, y): _____

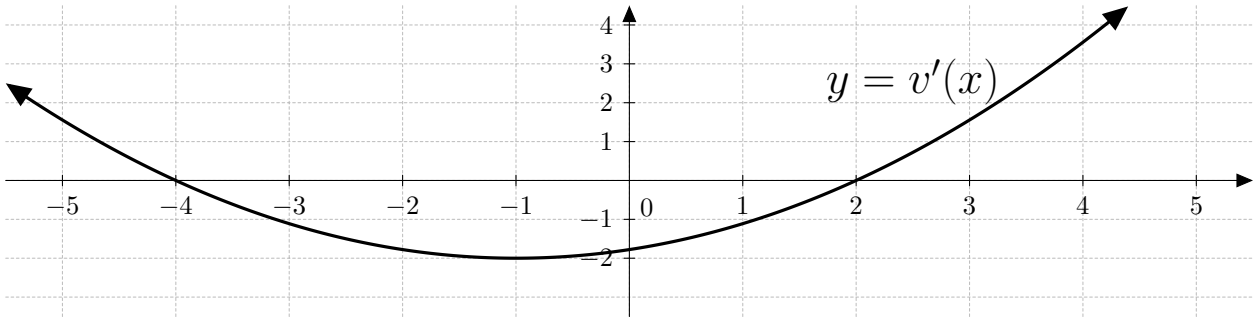
H. (1 point) Interval(s) $f(x)$ is concave up: _____

I. (1 point) Interval(s) $f(x)$ is concave down: _____

J. (1 point) Inflection point(s) (x, y): _____

K. (5 points) Sketch $y = f(x)$ on the graph below.





2. (3 points each) $y = v'(x)$ is plotted above. Find:

A. Interval(s) where $v(x)$ is increasing: _____ decreasing: _____

B. x-coordinate(s) where $v(x)$ has a local max: _____ local min: _____

C. Interval(s) where $v(x)$ is concave up: _____ concave down: _____

D. x-coordinate(s) where $v(x)$ has an inflection point: _____

3. (4 points each) In each of the following blanks, fill in "**max**" or "**min**".

A. If $h'(4) = 0$ and $h''(4) = 2$, then $h(x)$ has a local _____ at $x = 4$.

B. If $h'(-3) = 0$ and $h''(-3) = -2.7$, then $h(x)$ has a local _____ at $x = -3$.

4. (5 points) Find the differential dy if $y = \cos(x^2 + 3)$.

5. (10 points) Find the absolute maximum and absolute minimum of $w(x) = x^3 - 3x^2 + 1$ on $[-1, 1]$.

6. A. (7 points) Find the linearization of $g(x) = \sin(x)$ at $x = 0$.

B. (3 points) Use your answer from Part **A** to estimate $\sin(.01)$.

7. (5 points each) Find the following limits. (Use limit notation correctly.)

A. $\lim_{x\to\infty} \dfrac{e^x + 2}{x^2 + 4}$

B. $\lim_{x\to 1} \dfrac{\ln(x)}{x - 1}$

C. $\lim_{x\to-\infty} \dfrac{-5x^4 + x + 2}{7x^4 - x^3 - 2x + 1}$

D. $\lim_{x\to\infty} \dfrac{\sqrt{x^2 + 4}}{2x + 1}$

8. (10 points) Suppose that when your bakery sells cakes for x dollars each, your total profit is $P(x) = -x^2 + 20x - 75$ dollars. In order to maximize total profit, how much should your bakery charge per cake? (Make sure to justify why your answer corresponds to the absolute maximum.)

9. (10 points) A homeowner with 20 feet of fencing wants to enclose a rectangular area against the side of her house. What dimensions will maximize the fenced-in area? (Note that three sides of the rectangle will be formed from fencing, and the house will serve as the fourth side of the rectangle. Make sure to justify why your answer corresponds to the absolute maximum.)