Name Solutions Rec. Instr.

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Math 220

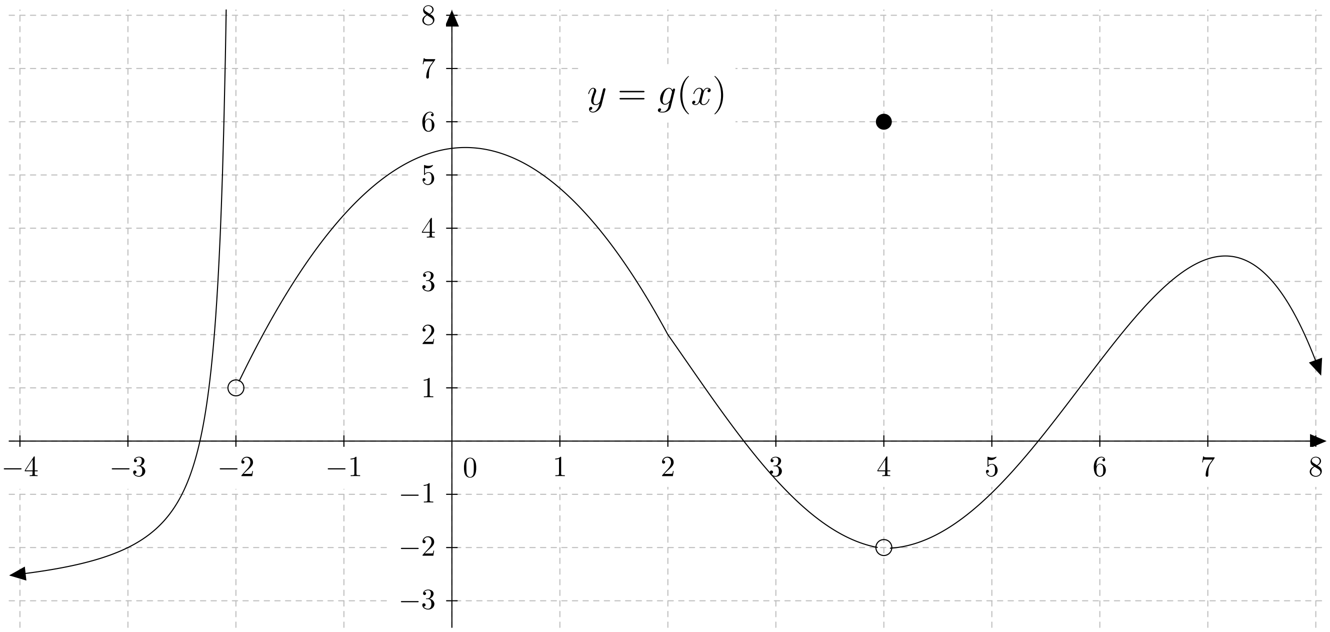
Exam 1

September 14, 2023

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam.

SHOW YOUR WORK!

Problem	Points	Points Possible	Problem	Points	Points Possible
1		16	8		6
2		2	9		6
3		4	10		4
4		6	11		6
5		6	12		6
6		6	13		20
7		12	Total Score		100



1. (2 points each) Consider the graph of $y = g(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x \to -2^-} g(x) = \infty$

B. $\lim_{x \to -2^+} g(x) = 1$

C. $\lim_{x \to -2} g(x)$ does not exist

D. $g(-2)$ does not exist

E. $\lim_{x \to 4^-} g(x) = -2$

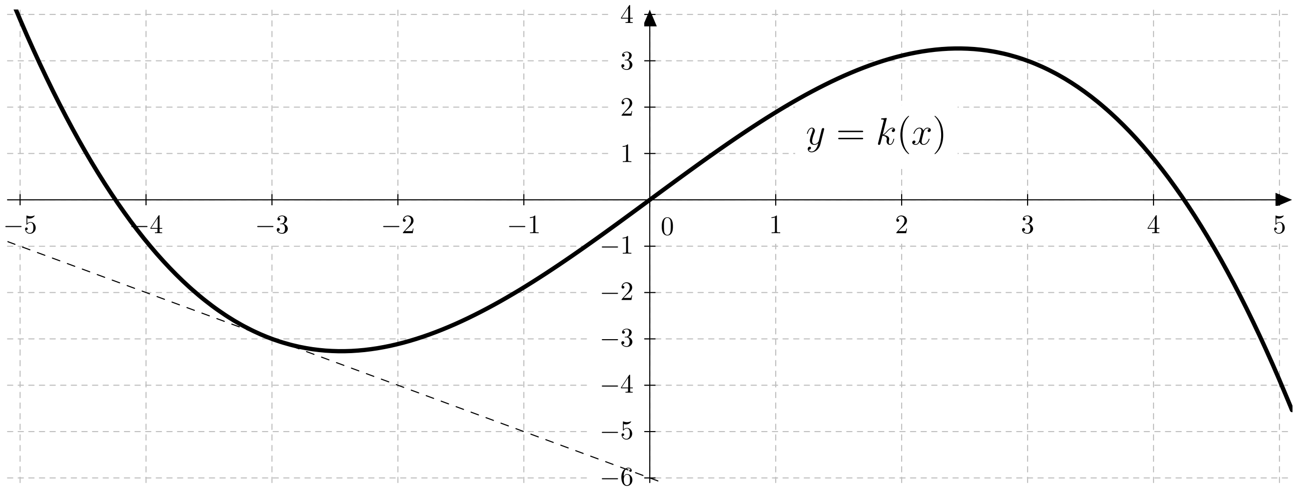
F. $\lim_{x \to 4^+} g(x) = -2$

G. $\lim_{x \to 4} g(x) = -2$

H. $g(4) = 6$

2. (2 points) Consider the graph of $y = g(x)$ above. List the x-coordinates where the function is discontinuous.

$x = -2$ and 4



3. (2 points each) The function $y = k(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values.

A. $k(-3) =$ -3

B. $k'(-3) =$ -1

4. (3 points each) Evaluate the following limits.

A. $\lim_{t\to 0} \dfrac{e^t}{t+2}$ $= \dfrac{e^0}{0+2} = \dfrac{1}{2}$

B. $\lim_{\theta\to 0} \dfrac{19\sin(\theta)}{\theta}$ $= 19\left(\lim_{\theta\to 0} \dfrac{\sin(\theta)}{\theta}\right) = 19 \cdot 1 = 19$

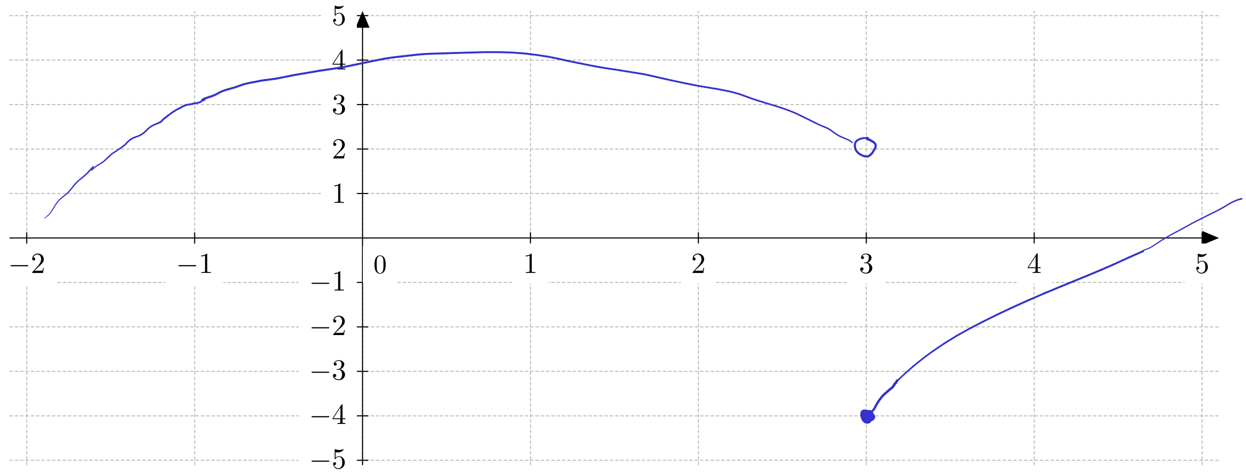
5. (3 points each) Given that $\lim_{x\to 6} u(x) = 4$ and $\lim_{x\to 6} w(x) = 5$, find the following limits.

A. $\lim_{x\to 6} \dfrac{w(x)+1}{u(x)}$ $= \dfrac{5+1}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

B. $\lim_{x\to 6} \sqrt{u(x)+w(x)}$ $= \sqrt{4+5} = \sqrt{9} = 3$

6. (6 points) Sketch the graph of $y = h(x)$ for a function that satisfies

$\lim_{x\to -1} h(x) = 3$, $\lim_{x\to 3^-} h(x) = 2$, and $\lim_{x\to 3^+} h(x) = -4$.



7. (6 points each) Evaluate the following limits.

A. $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x\to 2} (x+2)$

$= 2+2 = 4$

B. $\lim_{x\to 2} \left(\frac{1}{x-2} - \frac{2}{x^2-2x}\right) = \lim_{x\to 2} \left(\frac{1}{x-2} - \frac{2}{x(x-2)}\right)$

$= \lim_{x\to 2} \left(\frac{x}{x(x-2)} - \frac{2}{x(x-2)}\right) = \lim_{x\to 2} \frac{\cancel{x-2}}{x\,\cancel{(x-2)}}$

$= \lim_{x\to 2} \frac{1}{x} = \frac{1}{2}$

8. (6 points) Find $\lim_{x \to 2} m(x)$ provided that the function $m(x)$ satisfies $4x - 4 \leq m(x) \leq x^2$ for all $x \neq 2$. (**Justify your reasoning, and state the name of any theorem used.**)

$$\lim_{x \to 2} (4x-4) = 4 \cdot 2 - 4 = 8 - 4 = 4$$

$$\lim_{x \to 2} x^2 = 2^2 = 4$$

By the Squeeze Theorem, $\lim_{x \to 2} m(x) = 4$

9. The height in feet of a ball t seconds after being thrown directly upward is given by $y(t) = -16t^2 + 40t + 6$.

A. (4 points) Find the velocity 1 second after the ball is thrown (**include the units**).

$$y'(t) = -16 \cdot 2t + 40 = -32t + 40$$

$$y'(1) = -32 \cdot 1 + 40 = 8 \text{ ft/s}$$

B. (2 points) Is the ball going upward or downward 1 second after being thrown?

Upward because $y'(1) > 0$.

10. (4 points) Given that $n(7) = 3$ and $n'(x) = 2x - 12$, find the equation of the tangent line to the graph of $y = n(x)$ at $x = 7$.

$$n'(7) = 2 \cdot 7 - 12 = 2$$

$$y - 3 = 2(x - 7)$$

11. (6 points) Let $f(x) = \sqrt{x}$. **Using the limit definition of the derivative**, find $f'(9)$.

$$f'(9) = \lim_{h\to 0} \frac{f(9+h) - f(9)}{h} = \lim_{h\to 0} \frac{\sqrt{9+h} - \sqrt{9}}{h} = \lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h} \cdot \frac{-\sqrt{9+h} - 3}{-\sqrt{9+h} - 3} = \lim_{h\to 0} \frac{-(\cancel{9}+h) + \cancel{3\sqrt{9+h}} - \cancel{3\sqrt{9+h}} + \cancel{9}}{h(-\sqrt{9+h} - 3)}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(-\sqrt{9+h} - 3)} = \lim_{h\to 0} \frac{-1}{-\sqrt{9+h} - 3} = \frac{-1}{-\sqrt{9+0} - 3} = \frac{-1}{-6} = \frac{1}{6}$$

12. (6 points) Let $v(x) = x^2 - 5x$. **Using the limit definition of the derivative**, find $v'(x)$.

$$v'(x) = \lim_{h\to 0} \frac{v(x+h) - v(x)}{h} = \lim_{h\to 0} \frac{((x+h)^2 - 5(x+h)) - (x^2 - 5x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{5x} - 5h - \cancel{x^2} + \cancel{5x}}{h} = \lim_{h\to 0} \frac{2xh + h^2 - 5h}{h}$$

$$= \lim_{h\to 0} (2x + h - 5) = 2x + 0 - 5 = 2x - 5$$

13. (5 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\frac{d}{dx}\left(7x^3 - 3x + \pi^2 + e\right)$ $= 21x^2 - 3$

B. $\frac{d}{dx}\left(\frac{1}{x} + \sqrt{x} - 8\right)$ $= \frac{d}{dx}\left(x^{-1} + x^{1/2} - 8\right) = -x^{-2} + \frac{1}{2}x^{-1/2}$

C. $\frac{d}{dt}\left((t^2 - 7t + 2)(3t^5 - t^{-2})\right)$

$= \left[\frac{d}{dt}(t^2 - 7t + 2)\right](3t^5 - t^{-2}) + (t^2 - 7t + 2)\left[\frac{d}{dt}(3t^5 - t^{-2})\right]$
$= (2t - 7)\cdot(3t^5 - t^{-2}) + (t^2 - 7t + 2)(15t^4 + 2t^{-3})$

D. $\frac{d}{dx}\left(\frac{3x^3 - 5x}{6 + x^2}\right)$ $= \frac{\left[\frac{d}{dx}(3x^3 - 5x)\right]\cdot(6 + x^2) - (3x^3 - 5x)\left[\frac{d}{dx}(6 + x^2)\right]}{(6 + x^2)^2}$

$= \frac{(9x^2 - 5)(6 + x^2) - (3x^3 - 5x)\cdot 2x}{(6 + x^2)^2}$