Name Solutions Rec. Instr.
Signature Rec. Time

Math 220
Exam 2
October 12, 2023

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		20	6		6
2		10	7		7
3		10	8		8
4		10	9		9
5		10	10		10
			Total Score		100

1. (5 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\dfrac{d}{dx}\left(\ln(x)\cdot\arctan(x^5)\right) = \dfrac{1}{x}\cdot\arctan(x^5)+\ln(x)\cdot\dfrac{1}{1+(x^5)^2}\cdot 5x^4$

$$= \frac{\arctan(x^5)}{x} + \frac{5\cdot\ln(x)\cdot x^4}{1+x^{10}}$$

B. $\dfrac{d}{dx}\left(\dfrac{2^x}{e^x+1}\right) = \dfrac{2^x\cdot\ln(2)\cdot(e^x+1) - 2^x\cdot e^x}{(e^x+1)^2}$

C. $\dfrac{d^2}{d\theta^2}\tan(\theta) = \dfrac{d}{d\theta}\sec^2(\theta) = 2\sec(\theta)\left[\dfrac{d}{d\theta}\sec(\theta)\right]$

$$= 2\sec(\theta)\cdot\sec(\theta)\tan(\theta) = 2\sec^2(\theta)\tan(\theta)$$

D. $\dfrac{d}{d\theta}\cos\left(\sin(2\theta^3)\right) = -\sin\left(\sin(2\theta^3)\right)\left[\dfrac{d}{d\theta}\sin(2\theta^3)\right]$

$$= -\sin(\sin(2\theta^3))\cos(2\theta^3)\left[\frac{d}{d\theta}2\theta^3\right]$$

$$= -\sin(\sin(2\theta^3))\cos(2\theta^3)\cdot 6\theta^2$$

2. A. (7 points) Find the linearization of $g(x) = \ln(x)$ at $x = 1$.

$g'(x) = \frac{1}{x}$ So the linearization of $g(x)$ at $x=1$ is

$L(x) = g(1) + g'(1)(x-1) = \ln(1) + \frac{1}{1}(x-1) = x-1$

B. (3 points) Use your answer from Part **A** to estimate $\ln(1.07)$.

$\ln(1.07) = g(1.07) \approx L(1.07) = 1.07 - 1 = .07$

1.07 is close to 1

3. (10 points) Find the absolute maximum and absolute minimum of $w(x) = 2x^3 + 3x^2 - 12x + 1$ on $[0, 2]$.

$w'(x) = 6x^2 + 6x - 12 = 6(x^2+x-2) = 6(x+2)(x-1)$ is always defined. $w'(x)=0$ when $x=1$ or -2. The only critical number in $[0,2]$ is $x=1$.

$w(0) = 2\cdot 0^3 + 3\cdot 0^2 - 12\cdot 0 + 1 = 1$

$w(1) = 2\cdot 1^3 + 3\cdot 1^2 - 12\cdot 1 + 1 = -6$

$w(2) = 2\cdot 2^3 + 3\cdot 2^2 - 12\cdot 2 + 1 = 5$

On $[0,2]$, $w(x)$ has an absolute max $w(2)=5$ and an absolute min $w(1)=-6$.

4. (10 points) Find the derivative of $h(x) = x^{7x^2}$.

$$\ln(h(x)) = \ln\left(x^{7x^2}\right) = 7x^2 \ln(x)$$

$$\frac{d}{dx} \ln(h(x)) = \frac{d}{dx}\left(7x^2 \ln(x)\right)$$

$$\frac{h'(x)}{h(x)} = 14x \cdot \ln(x) + 7x^2 \cdot \frac{1}{x} = 14x \cdot \ln(x) + 7x$$

$$h'(x) = h(x)\left(14x \ln(x) + 7x\right)$$

$$h'(x) = x^{7x^2}\left(14x \ln(x) + 7x\right)$$

5. (10 points) If $\sin(xy^2) = x^2$, compute $\dfrac{dy}{dx}$ in terms of x and y.

$$\frac{d}{dx} \sin(xy^2) = \frac{d}{dx} x^2$$

$$\cos(xy^2)\left[\frac{d}{dx}(xy^2)\right] = 2x$$

$$\cos(xy^2)\left(1 \cdot y^2 + x \cdot 2y \frac{dy}{dx}\right) = 2x$$

$$\cos(xy^2) \cdot y^2 + \cos(xy^2) \cdot 2xy \frac{dy}{dx} = 2x$$

$$\cos(xy^2) \cdot 2xy \frac{dy}{dx} = 2x - \cos(xy^2) \cdot y^2$$

$$\frac{dy}{dx} = \frac{2x - \cos(xy^2) \cdot y^2}{\cos(xy^2) \cdot 2xy}$$

6. (6 points) The surface area of a sphere is $A = 4\pi r^2$, where r denotes the radius of the sphere. Find a formula for the differential dA in terms of r and dr.

$$\frac{dA}{dr} = 8\pi r$$

$$dA = 8\pi r\,dr$$

7. (7 points) Suppose that the position of an object is given by $s(t) = e^{t^3-t}$ meters at time t seconds. Find the instantaneous velocity of the object at time $t = 1$ second. (Include units with your answer.)

$$s'(t) = e^{t^3-t} \cdot \left[\frac{d}{dt}(t^3-t)\right] = e^{t^3-t}(3t^2-1)$$

$$s'(1) = e^{1^3-1} \cdot (3 \cdot 1^2 - 1) = e^0 \cdot 2 = 2 \text{ m/s}$$

8. (8 points) Let $p(t)$ denote the position of a particle in cm after t seconds. Suppose that we are unable to write down a nice formula for $p(t)$, but we happen to know that when $t = 5$ seconds, the particle has position 3 cm and velocity -2 cm/s. Find the linearization of $p(t)$ at $t = 5$ seconds, and use it to approximate the location of the particle when $t = 5.2$ seconds. (Include units with your answer.)

The linearization of $p(t)$ at $t=5$ is

$$L(t) = p(5) + p'(5) \cdot (t-5) = 3 - 2(t-5)$$

$$p(5.2) \underset{\substack{\uparrow \\ \text{5.2 is close to 5}}}{\approx} L(5.2) = 3 - 2(5.2-5) = 3 - 2 \cdot (.2) = 2.6 \text{ cm}$$

9. (9 points) The volume of a sphere is $V = \dfrac{4\pi r^3}{3}$, where r denotes the radius of the sphere. If the radius of a sphere is increasing at a constant rate of $\dfrac{1}{2}$ m/s, find the rate at which the volume is increasing when the radius is 2 m. (Include units with your answer.)

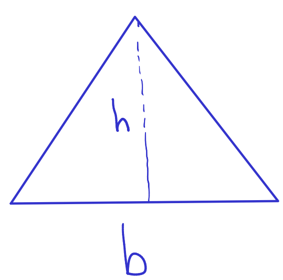
Want: $\frac{dV}{dt}$ when $r = 2m$

Know: $\frac{dr}{dt} = \frac{1}{2}$ m/s

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4\pi r^3}{3}\right) = \frac{4\pi}{3}\cdot 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

When $r = 2m$, $\frac{dV}{dt} = 4\pi \cdot 2^2 \cdot \left(\frac{1}{2}\right) = 8\pi$ m³/s

10. (10 points) The base of a triangle is increasing at a rate of 5 ft/s, and the height of the triangle is increasing at a rate of 2 ft/s. Find the rate at which the area of the triangle is changing when the base is 3 ft and the height is 4 ft. (Include units with your answer.)



Want: $\frac{dA}{dt}$ when $b = 3$ ft and $h = 4$ ft.

Know: $\frac{db}{dt} = 5$ ft/s and $\frac{dh}{dt} = 2$ ft/s

$A = \frac{1}{2}bh$

$$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2}bh\right) = \frac{1}{2}\cdot\frac{db}{dt}\cdot h + \frac{1}{2}\cdot b\cdot\frac{dh}{dt}$$

When $b = 3$ ft and $h = 4$ ft, we have

$$\frac{dA}{dt} = \frac{1}{2}\cdot 5\cdot 4 + \frac{1}{2}\cdot 3\cdot 2 = 10 + 3 = 13 \text{ ft}^2/\text{s}$$