Name Solutions Rec. Instr.

Signature Rec. Time

Math 220
Exam 3
November 9, 2023

No books, calculators, or notes are allowed. *Please make sure that all cell phones, laptops, tablets, and smartwatches are turned off and put away.* You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		16	6		12
2		25	7		8
3		8	8		10
4		6	9		8
5		7	Total Score		100

1. The function $f(x)$ and its first and second derivatives are:

$$f(x) = \frac{12}{x^2+3} \qquad f'(x) = \frac{-24x}{(x^2+3)^2} \qquad f''(x) = \frac{72(x-1)(x+1)}{(x^2+3)^3}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: $(-\infty, \infty)$

B. (1 point) y-intercept: $f(0) = \frac{12}{0^2+3} = 4$

C. (1 point) x-intercept(s): none

D. (1 point) Horizontal asymptote(s): $y = 0$

$$\lim_{x\to\infty} \frac{12}{x^2+3} = 0 \qquad \lim_{x\to-\infty} \frac{12}{x^2+3} = 0$$

E. (1 point) Interval(s) $f(x)$ is increasing: $(-\infty, 0)$

F. (1 point) Interval(s) $f(x)$ is decreasing: $(0, \infty)$

$f'(x) = 0$ at $x = 0$

f(x)	↗		↘
sign of f(x)	+		−
		0	

G. (1 point) Local maximum(s) (x, y): $(0, 4)$

H. (1 point) Local minimum(s) (x, y): none

I. (1 point) Interval(s) $f(x)$ is concave up: $(-\infty, -1)$ and $(1, \infty)$

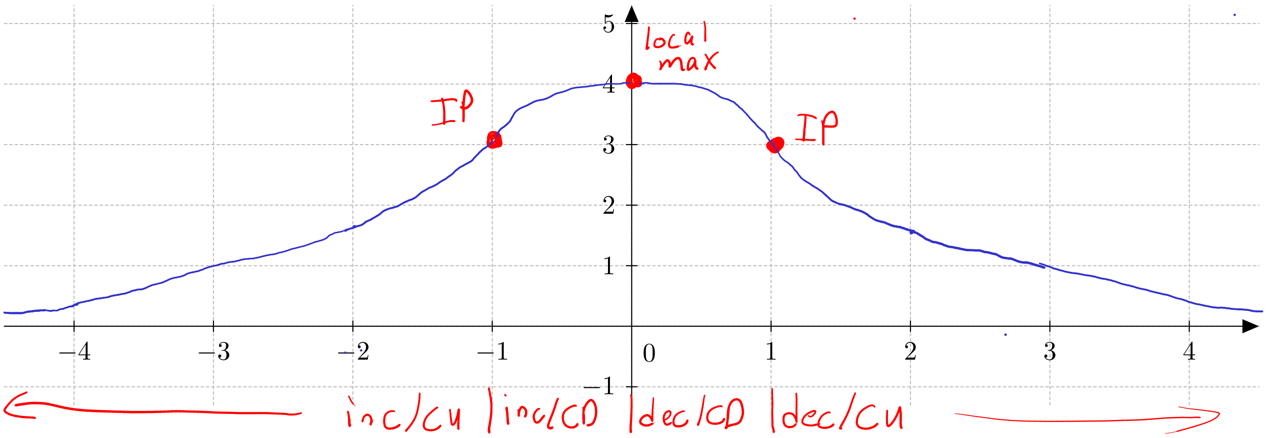
J. (1 point) Interval(s) $f(x)$ is concave down: $(-1, 1)$

$f''(x) = 0 \iff x = \pm 1$

f(x)	CU		CD		CU
sign of f''(x)	+		−		+
		−1		1	

K. (1 point) Inflection point(s) (x, y): $(-1, 3)$ and $(1, 3)$

L. (5 points) Sketch $y = f(x)$ on the graph below.



2. (5 points each) Evaluate the following:

A. $$\lim_{x\to\infty} \frac{6x^7-3x^2+x+1}{-5x^7+2x^3+8} = \frac{6}{-5} = -\frac{6}{5}$$

B. $$\lim_{\theta\to 0}\frac{\sin(\theta^2)+\theta^2}{\theta^2} \underset{\left(\substack{LH \\ \frac{0}{0}}\right)}{=} \lim_{\theta\to 0}\frac{\cos(\theta^2)\cdot 2\theta+2\theta}{2\theta} = \lim_{\theta\to 0}\frac{\cancel{2\theta}(\cos(\theta^2)+1)}{\cancel{2\theta}}$$

$$= \lim_{\theta\to 0}\left(\cos(\theta^2)+1\right) = \cos(0^2)+1 = 1+1 = 2$$

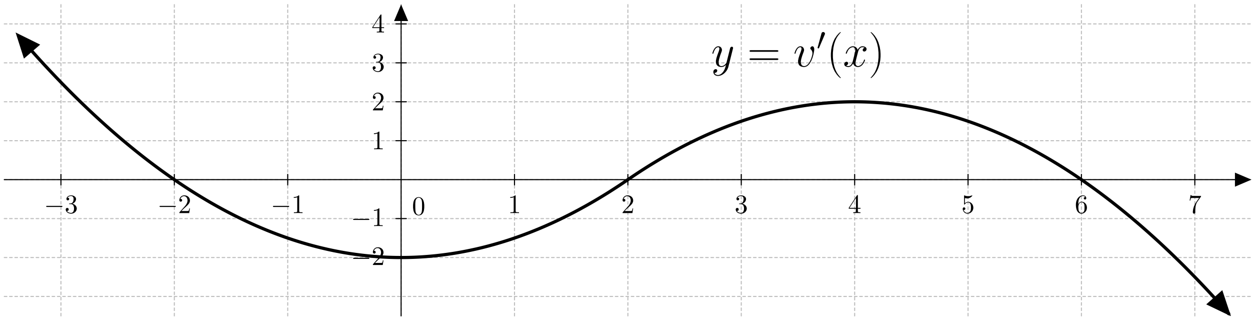
C. $$\lim_{x\to\infty}\frac{x\ln(x)+3}{e^x+x} \underset{\left(\substack{LH \\ \frac{\infty}{\infty}}\right)}{=} \lim_{x\to\infty}\frac{1\cdot\ln(x)+x\cdot\frac{1}{x}}{e^x+1} = \lim_{x\to\infty}\frac{\ln(x)+1}{e^x+1}$$

$$\underset{\left(\substack{LH \\ \frac{\infty}{\infty}}\right)}{=} \lim_{x\to\infty}\frac{\frac{1}{x}}{e^x} = \lim_{x\to\infty}\frac{1}{xe^x} = 0$$

D. $$\int \left(x^{4/3}+7\sin(x)+3\cos(x)\right)\,dx = \frac{3}{7}x^{7/3} - 7\cos(x) + 3\sin(x) + C$$

E. $$\int_0^1 (e^x+2x)\,dx = \left(e^x+x^2\right)\Big|_0^1 = (e^1+1^2)-(e^0+0^2)$$

$$= e+1-1 = e$$



3. (2 points each) $y = v'(x)$ is plotted above. Find:

A. Interval(s) where $v(x)$ is increasing: $(-\infty,-2),(2,6)$ decreasing: $(-2,2),(6,\infty)$

B. x-coordinate(s) where $v(x)$ has a local max: -2 and 6 local min: 2

C. Interval(s) where $v(x)$ is concave up: $(0,4)$ concave down: $(-\infty,0),(4,\infty)$

D. x-coordinate(s) where $v(x)$ has an inflection point: 0 and 4

4. (3 points each) In each of the following blanks, fill in "**max**" or "**min**".

A. If $h'(5) = 0$ and $h''(5) = -2$, then $h(x)$ has a local max at $x = 5$.

B. If $h'(-1) = 0$ and $h''(-1) = 6$, then $h(x)$ has a local min at $x = -1$.

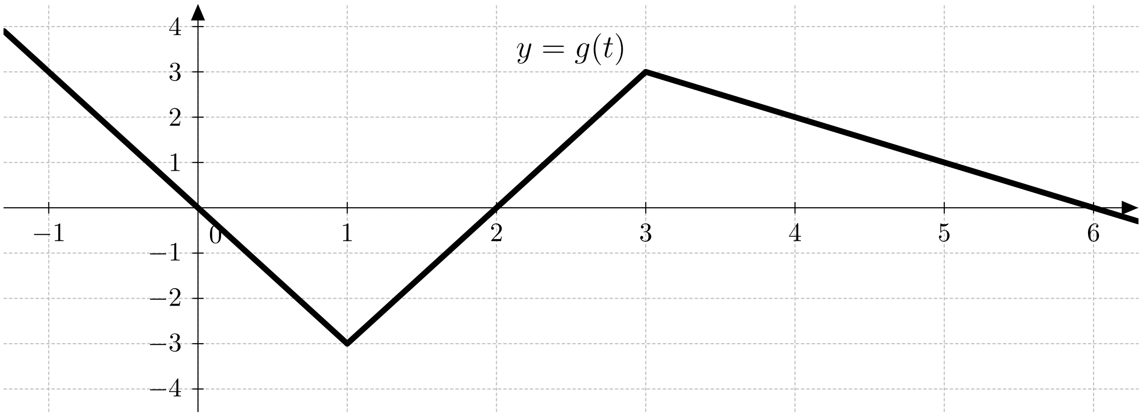
5. (7 points) Find $\dfrac{d}{dx}\displaystyle\int_0^{x^3} \sin(t^2)\,dt$.

$\int_0^{x^3} \sin(t^2)dt = f(g(x))$ where $f(x) = \int_0^{x} \sin(t^2)dt$ and $g(x) = x^3$.

$f'(x) = \sin(x^2)$ and $g'(x) = 3x^2$

$\frac{d}{dx}\int_0^{x^3} \sin(t^2)dt = \frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

$= \sin((x^3)^2)\cdot 3x^2 = 3x^2 \sin(x^6)$



6. (4 points each) $y = g(t)$ is plotted above. Let $A(x) = \displaystyle\int_0^x g(t)\,dt$. Find the following quantities.

A. $A(2) =$ $\int_0^2 g(t)\,dt = -\frac{1}{2}\cdot 2\cdot 3 = -3$

B. $A(-1) =$ $\int_0^{-1} g(t)\,dt = -\int_{-1}^{0} g(t)\,dt = -\frac{1}{2}\cdot 1\cdot 3 = -\frac{3}{2}$

C. $A'(4) =$ $g(4) = 2$ because $A'(x) = \frac{d}{dx}\int_0^x g(t)\,dt = g(x)$

7. (8 points) Find $w(x)$ if $w'(x) = 3\sqrt{x} - 6x^2$ and $w(0) = 8$.

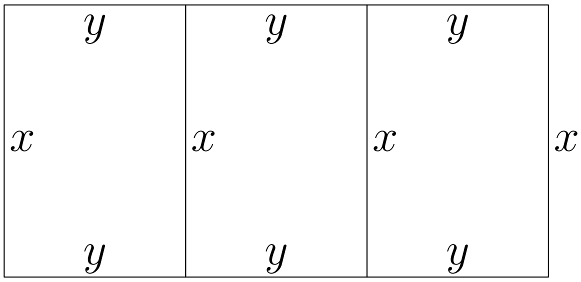
$w(x) = 3\cdot\frac{2}{3}x^{3/2} - 6\cdot\frac{1}{3}x^3 + C = 2x^{3/2} - 2x^3 + C$

for some constant C.

$8 = w(0) = 2\cdot 0^{3/2} - 2\cdot 0^3 + C = C$ so

$w(x) = 2x^{3/2} - 2x^3 + 8$

8. (10 points) Suppose that you have 24 meters of fencing to make three adjacent rectangular kennels of length x meters and width y meters (see the diagram below). Find the values of x and y that maximize the enclosed area. (Justify why your answer corresponds to an absolute maximum, and include units in your answer.)



Maximize Area $A=3xy$

$6y+4x=24$

$6y=24-4x$

$y=\frac{24-4x}{6}=4-\frac{2}{3}x$

Maximize $A(x)=3x(4-\frac{2}{3}x)=12x-2x^2$ on $(0,\infty)$

$A'(x)=12-4x$ is always defined, and $A'(x)=0 \iff 12-4x=0$

$\iff 12=4x$

$\iff 3=x$

A(x) ↗ ↘

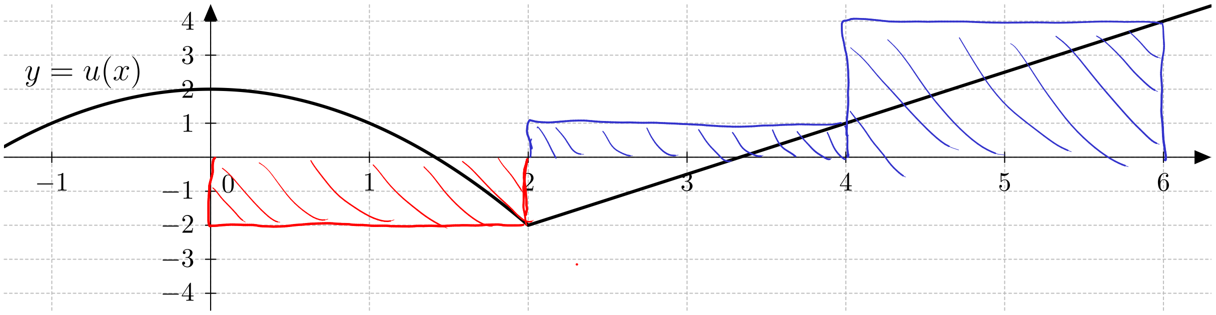
sign of $A'(x)$ + −

0 3

$\boxed{\text{or}}$

$A''(x)=-4$ so $A(x)$ is always concave down

The area is maximized when $x=3m$ and $y=4-\frac{2}{3}\cdot 3=2m$.



9. (8 points) Estimate $\int_0^6 u(x)\,dx$ by computing R_3, the Right-Endpoint Approximation with 3 subintervals. Also, illustrate the rectangles on the graph above.

$\Delta x=\frac{6-0}{3}=2$

$\int_0^6 u(x)\,dx \approx R_3 = u(2)\cdot\Delta x + u(4)\cdot\Delta x + u(6)\cdot\Delta x$

$= -2\cdot 2 + 1\cdot 2 + 4\cdot 2 = -4+2+8$

$= 6$