Name _____ Rec. Instr._____
Signature _____ Rec. Time_____

Math 220
Exam 3
November 9, 2023

No books, calculators, or notes are allowed. *Please make sure that all cell phones, laptops, tablets, and smartwatches are turned off and put away.* You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		16	6		12
2		25	7		8
3		8	8		10
4		6	9		8
5		7	Total Score		100

1. The function $f(x)$ and its first and second derivatives are:

$$f(x) = \frac{12}{x^2+3} \qquad f'(x) = \frac{-24x}{(x^2+3)^2} \qquad f''(x) = \frac{72(x-1)(x+1)}{(x^2+3)^3}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: _____

B. (1 point) y-intercept: _____

C. (1 point) x-intercept(s): _____

D. (1 point) Horizontal asymptote(s): _____

E. (1 point) Interval(s) $f(x)$ is increasing: _____

F. (1 point) Interval(s) $f(x)$ is decreasing: _____

G. (1 point) Local maximum(s) (x, y): _____

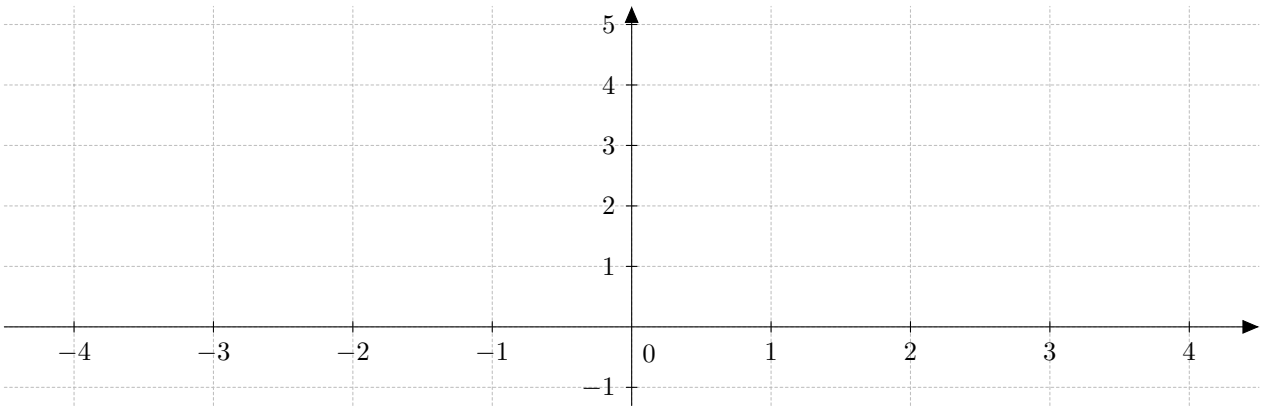
H. (1 point) Local minimum(s) (x, y): _____

I. (1 point) Interval(s) $f(x)$ is concave up: _____

J. (1 point) Interval(s) $f(x)$ is concave down: _____

K. (1 point) Inflection point(s) (x, y): _____

L. (5 points) Sketch $y = f(x)$ on the graph below.



2. (5 points each) Evaluate the following:

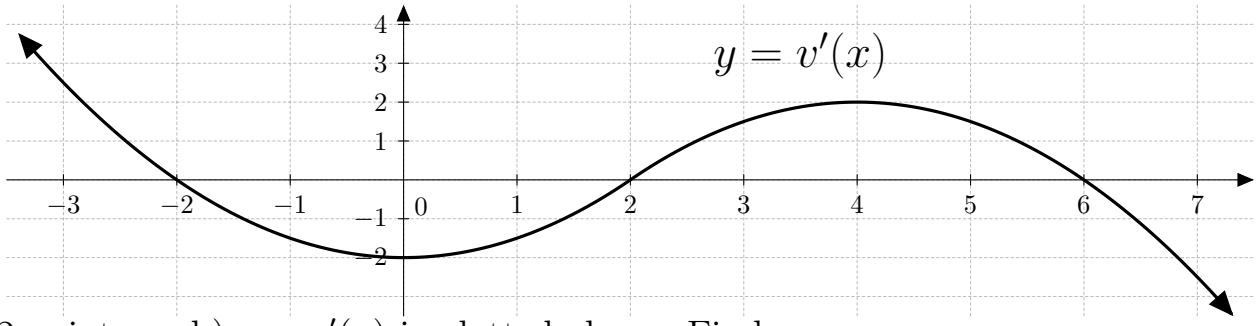
A. $\lim_{x\to\infty} \dfrac{6x^7 - 3x^2 + x + 1}{-5x^7 + 2x^3 + 8}$

B. $\lim_{\theta\to 0} \dfrac{\sin(\theta^2) + \theta^2}{\theta^2} =$

C. $\lim_{x\to\infty} \dfrac{x\ln(x) + 3}{e^x + x} =$

D. $\int \left(x^{4/3} + 7\sin(x) + 3\cos(x)\right)\, dx =$

E. $\int_0^1 (e^x + 2x)\, dx \;=$



3. (2 points each) $y = v'(x)$ is plotted above. Find:

A. Interval(s) where $v(x)$ is increasing: _____ decreasing: _____

B. x-coordinate(s) where $v(x)$ has a local max: _____ local min: _____

C. Interval(s) where $v(x)$ is concave up: _____ concave down: _____

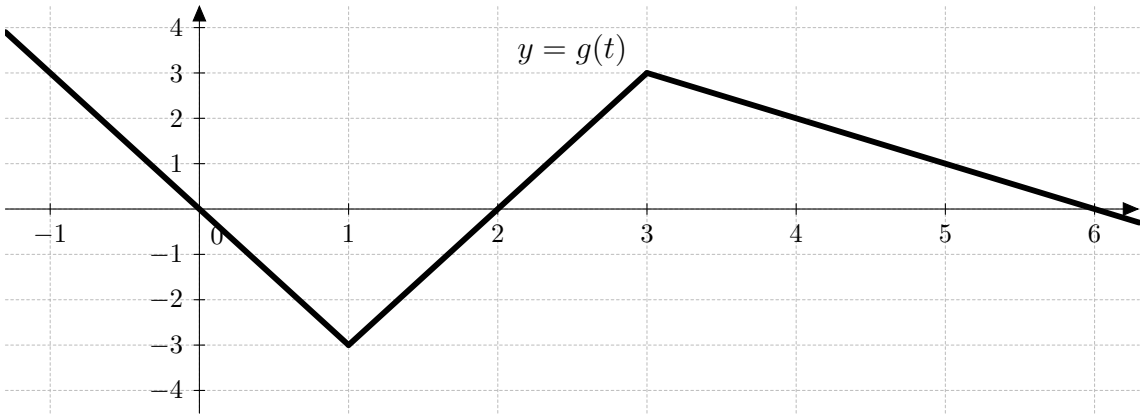
D. x-coordinate(s) where $v(x)$ has an inflection point: _____

4. (3 points each) In each of the following blanks, fill in "**max**" or "**min**".

A. If $h'(5) = 0$ and $h''(5) = -2$, then $h(x)$ has a local _____ at $x = 5$.

B. If $h'(-1) = 0$ and $h''(-1) = 6$, then $h(x)$ has a local _____ at $x = -1$.

5. (7 points) Find $\dfrac{d}{dx}\displaystyle\int_0^{x^3} \sin(t^2)\,dt.$



6. (4 points each) $y = g(t)$ is plotted above. Let $A(x) = \int_0^x g(t)\,dt$. Find the following quantities.

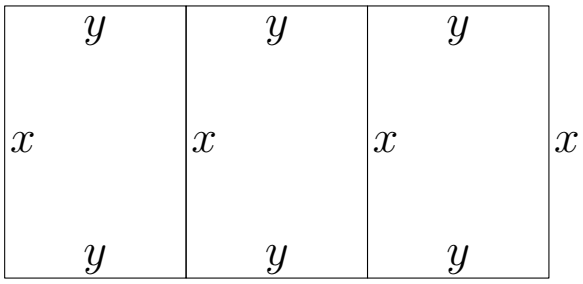
A. $A(2) =$

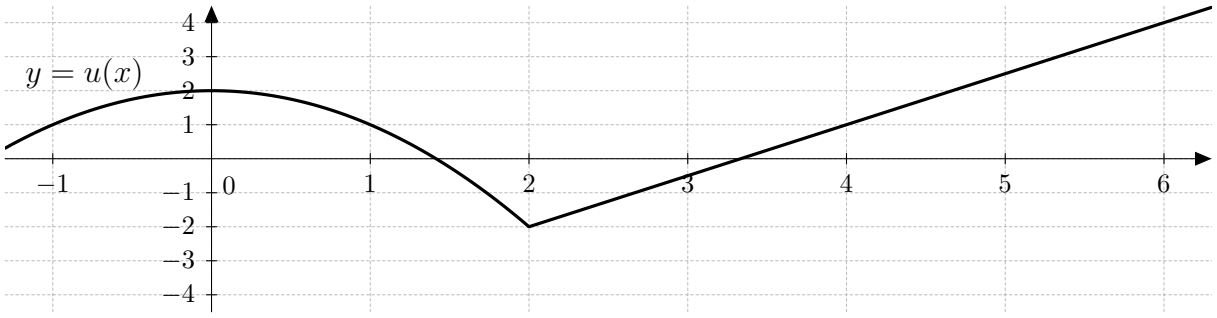
B. $A(-1) =$

C. $A'(4) =$

7. (8 points) Find $w(x)$ if $w'(x) = 3\sqrt{x} - 6x^2$ and $w(0) = 8$.

8. (10 points) Suppose that you have 24 meters of fencing to make three adjacent rectangular kennels of length x meters and width y meters (see the diagram below). Find the values of x and y that maximize the enclosed area. (Justify why your answer corresponds to an absolute maximum, and include units in your answer.)





9. (8 points) Estimate $\int_0^6 u(x)\,dx$ by computing R_3, the Right-Endpoint Approximation with 3 subintervals. Also, illustrate the rectangles on the graph above.