Name Solutions Rec. Instr.

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Math 220

Exam 2

March 2, 2023

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, show your work on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		30	6		5
2		10	7		10
3		9	8		9
4		9	9		9
5		9	**Total Score**		100

1. (6 points each) Find the following derivatives. You **do not need to simplify** your answers or show your work. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\frac{d}{dx}\left(\frac{6}{x^2}+6^x+e^2\right) = 6\cdot(-2)x^{-3}+6^x\ln(6) = -\frac{12}{x^3}+6^x\cdot\ln(6)$

B. $\frac{d}{dx}\left(x^{5/2}\cdot\ln(x)\right) = \frac{5}{2}x^{3/2}\cdot\ln(x)+x^{5/2}\cdot\frac{1}{x} = \frac{5}{2}x^{3/2}\cdot\ln(x)+x^{3/2}$

C. $\frac{d}{dt}\arctan(t^3-5t) = \frac{1}{1+(t^3-5t)^2}\cdot(3t^2-5) = \frac{3t^2-5}{1+(t^3-5t)^2}$

D. $\frac{d}{d\theta}\sqrt{\cos(5\theta)} = \frac{1}{2\sqrt{\cos(5\theta)}}\cdot(-\sin(5\theta))\cdot 5 = \frac{-5\sin(5\theta)}{2\sqrt{\cos(5\theta)}}$

E. $\frac{d}{dx}\left(\frac{\tan(x)-x}{e^x+x}\right) = \frac{(\sec^2(x)-1)(e^x+x)-(\tan(x)-x)(e^x+1)}{(e^x+x)^2}$

2. (10 points) Find the equation of the tangent line to the curve $y = \sin(x) + 2$ at $x = 0$.

$$\frac{dy}{dx} = \cos(x)$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \cos(0) = 1$$

The tangent line at $x=0$ has slope 1 and goes through $(0, \sin(0)+2) = (0,2)$ so it is

$$y - 2 = 1\cdot(x-0)$$

$$\left(\text{or } y - 2 = x\right)$$

$$\left(\text{or } y = x + 2\right)$$

3. (9 points) Let $g(x) = 7x^2 + x$. Using the **limit definition of the derivative**, find $g'(x)$. Make sure to use limit notation correctly.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h\to 0} \frac{(7(x+h)^2 + (x+h)) - (7x^2 + x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{7x^2} + 14xh + 7h^2 \cancel{+x} + h \cancel{-7x^2} \cancel{-x}}{h}$$

$$= \lim_{h\to 0} \frac{14xh + 7h^2 + h}{h} = \lim_{h\to 0} (14x + 7h + 1)$$

$$= 14x + 7\cdot 0 + 1 = 14x + 1$$

4. (9 points) Find $\dfrac{dy}{dx}$ if $x^4 + xy + 2y^3 = 7$.

$$\frac{d}{dx}(x^4 + xy + 2y^3) = \frac{d}{dx} 7$$

$$4x^3 + 1\cdot y + x\frac{dy}{dx} + 2\cdot 3y^2\frac{dy}{dx} = 0$$

$$4x^3 + y + x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = -4x^3 - y$$

$$(x + 6y^2)\frac{dy}{dx} = -4x^3 - y$$

$$\frac{dy}{dx} = \frac{-4x^3 - y}{x + 6y^2}$$

5. (9 points) Find $\dfrac{dy}{dx}$ if $\sin(xy) = x^2$.

$$\frac{d}{dx}\sin(xy) = \frac{d}{dx}x^2$$

$$\cos(xy)\left(1\cdot y + x\frac{dy}{dx}\right) = 2x$$

$$y + x\frac{dy}{dx} = \frac{2x}{\cos(xy)}$$

$$x\frac{dy}{dx} = \frac{2x}{\cos(xy)} - y$$

$$\frac{dy}{dx} = \frac{2}{\cos(xy)} - \frac{y}{x}$$

$\boxed{\text{or}}$

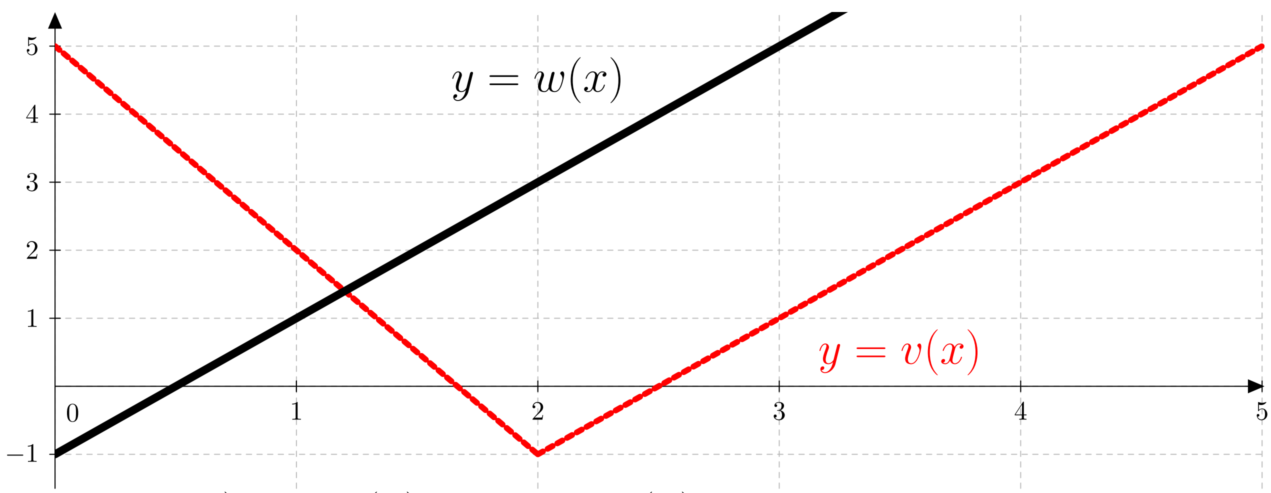
$$\cos(xy)\cdot y + \cos(xy)\cdot x\cdot\frac{dy}{dx} = 2x$$

$$\cos(xy)\,x\cdot\frac{dy}{dx} = 2x - \cos(xy)\cdot y$$

$$\frac{dy}{dx} = \frac{2x - \cos(xy)\cdot y}{\cos(xy)\cdot x}$$

6. (5 points) The height of a plane in meters at time t seconds after the plane elevates off of the ground at takeoff is given by $h(t)$. Is $h'(1)$ positive or negative? Briefly explain your answer.

$h'(1)$ is positive because the plane's altitude is increasing 1 second after takeoff.



7. (5 points each) $y = v(x)$ and $y = w(x)$ are graphed above. Suppose that $p(x) = v(x) \cdot w(x)$ and $q(x) = v(w(x))$. Find:

A. $p'(1)$

$$p'(x) = v'(x)w(x) + v(x)w'(x)$$
$$p'(1) = v'(1)w(1) + v(1)w'(1)$$
$$= -3 \cdot 1 + 2 \cdot 2$$
$$= 1$$

B. $q'(2)$

$$q'(x) = v'(w(x)) \cdot w'(x)$$
$$q'(2) = v'(w(2))\, w'(2)$$
$$= v'(3) \cdot 2$$
$$= 2 \cdot 2$$
$$= 4$$

8. (9 points) Find the derivative of $f(x) = x^{\cos(x)}$.

$$\ln(f(x)) = \ln\left(x^{\cos(x)}\right) = \cos(x) \cdot \ln(x)$$

$$\frac{d}{dx} \ln(f(x)) = \frac{d}{dx}\left(\cos(x) \cdot \ln(x)\right)$$

$$\frac{f'(x)}{f(x)} = -\sin(x) \cdot \ln(x) + \cos(x) \cdot \frac{1}{x}$$

$$f'(x) = f(x)\left(-\sin(x)\ln(x) + \frac{\cos(x)}{x}\right)$$

$$f'(x) = x^{\cos(x)}\left(-\sin(x)\ln(x) + \frac{\cos(x)}{x}\right)$$

9. The height in feet of a ball t seconds after being thrown directly upward is given by $y(t) = -16t^2 + 40t + 5$.

A. (6 points) Find the velocity 1 second after the ball is thrown (**include units with your answer**).

$$y'(t) = -32t + 40$$

$$y'(1) = -32 \cdot 1 + 40 = 8 \text{ ft/s}$$

B. (3 points) Is the ball going upward or downward 1 second after being thrown?

upward because $y'(1) > 0$